# Time Reversal in elastodynamics and applications to Structural Health Monitoring

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# Detection and Localization of Damage

- Important in SHM systems
- Response recordings at a number of sensors to monitor structural integrity [1]
- Detection : comparison of recordings to a reference (undamaged) state
- Localization : Inverse Problem usually ill-posed
- Solution : *Time-Reversal* (TR) computational tool introduced by Fink et. al. [2]
- Achieves refocusing of the wave on the source
- Sending back the recorded signals but reversed in time

1. GE Stavroulakis, (2000) Inverse and crack identification problems in engineering mechanics
2. Fink et. al., (2000) Time-reversed acoustics

# Time Reversal and applications

- TR is a physical process
- It exploits the time reversibility (based on spatial reciprocity and time reversal invariance) of linear wave equations
- Robust and Simple technique for source localization
- Has been applied in Acoustics [3], Elastodynamics [4], Electromagnetism, Hydrodynamics etc.
- Finds several applications in medicine, telecommunications, underwater acoustics, seismology, engineering structures, etc.
- Example of source localization in acoustic medium here

3. L Borcea, G Papanicolaou, C Tsogka and J Berryman, (2002) Imaging and time reversal in random media
4. D Givoli, (2014) Time Reversal as a Computational Tool in Acoustics and Elastodynamics

# Time Reversal

- Two step approach :
  - Forward step : waves emitted from some source, travel through the medium and the response is recorded by receivers
  - Backward step : the recorded signals are time reversed and retransmitted
- Ideal Conditions :
  - receivers over the entire domain or it's entire boundary
  - recordings of the field variable and its derivatives
  - recordings during the whole experiment time T
  - absence of noise
- Difficulty : the refocusing time is unknown
- Procedure for the assessment of the refocusing time (stopping criterion)

# TR for scatterer localization

- Scatterers act as secondary sources
- Emit at every passage of the original pulse
- Multiple in time sources
- Knowledge of the response in the reference (healthy) configuration
- Scattered field ($p^{scat} = p^{tot} - p^{ref}$) results better refocusing
- Example of defect localization in acoustic medium here

# In the present work

- Description of the numerical implementation of TR
- Elastic medium
- Bounded domain $\Omega$
- Excitation produced by $N_s$ point sources forming $\Omega_s \subset \Omega$
- Response recordings at $N_r$ receivers forming $\Omega_r \subset \Omega$
- $\Omega_r \cap \Omega_s = \emptyset$ or $\Omega_r \cap \Omega_s \neq \emptyset$ or $\underline{\Omega_r = \Omega_s}$
- Sensors may form an array or be distributed
- DORT method [5], [6] for selective refocusing on multiple defect using the SVD of the Impulse Response Matrix

5. G Derveaux, G Papanicolaou and C Tsogka, (2007) Time reversal imaging for sensor networks with optimal compensation in time
6. E Barbieri and M Meo, (2010) Time reversal DORT method applied to nonlinear elastic wave scattering

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# Forward step

- Simulated Numerically using a mixed finite element formulation [7]
- Wave propagation model

Displacement - Stress (second order)

$$\rho \frac{\partial^2 \boldsymbol{u}}{\partial t^2} - \operatorname{div} \sigma = \delta(\boldsymbol{x} - \boldsymbol{x}_s) f(t) \boldsymbol{e}_i$$

$$\sigma = C : \varepsilon$$

strain - displacement relationship

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Velocity - Stress (first order)

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} - \operatorname{div} \sigma = \delta(\boldsymbol{x} - \boldsymbol{x}_s) f(t) \boldsymbol{e}_i$$

$$A : \frac{\partial \sigma}{\partial t} - \dot{\varepsilon} = 0$$

strain rate - velocity relationship

$$\dot{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$$

- Homogeneous Dirichlet boundary conditions and zero initial conditions

7. E Bécache, P Joly and C Tsogka (2002) A new family of mixed finite elements for the linear elastodynamic problem.

# Backward step

- Always performed numerically in SHM applications
- Three alternative forms
  a) imposed displacements at all $\boldsymbol{x}_r$
  b) appropriate initial conditions
  c) sensors acting as sources introducing right hand side loading terms

$$\rho \frac{\partial \tilde{\boldsymbol{v}}}{\partial t} - \operatorname{div} \tilde{\sigma} = \sum_{q=1}^{N_r} \delta(\boldsymbol{x} - \boldsymbol{x}_q) \boldsymbol{v}(\boldsymbol{x}_q, T - t), \qquad (\boldsymbol{x}, t) \in \Omega \times (0, T]$$

$$A : \frac{\partial \tilde{\sigma}}{\partial t} - \dot{\tilde{\varepsilon}} = 0, \qquad (\boldsymbol{x}, t) \in \Omega \times (0, T]$$

$$\tilde{\boldsymbol{v}}(\boldsymbol{x}, t) = 0, \qquad (\boldsymbol{x}, t) \in \partial\Omega \times (0, T]$$

$$\tilde{\boldsymbol{v}}(\boldsymbol{x}, 0) = 0 \quad \text{and} \quad \tilde{\sigma}(\boldsymbol{x}, 0) = 0, \qquad \boldsymbol{x} \in \Omega$$

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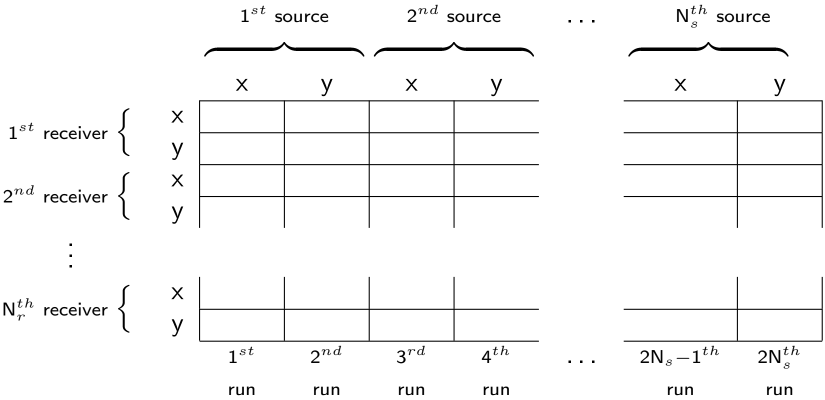
# Impulse Response Matrix (IRM)

- In every time step of the forward propagation



# Multiple damaged areas

- DORT method - imaging procedure for selective refocusing on different targets
- Fourier transform of the IRM $P(t)$ to obtain $\hat{P}(\omega)$
- Singular Value Decomposition (SVD) of $\hat{P}(\omega)$ according to

$$\hat{P}(\omega) = U(\omega)S(\omega)V^*(\omega)$$

- Projection of each column $\hat{P}^{(l)}$ of the transformed IRM on the $k$-th singular vector as

$$\hat{P}_k^{(p)}(\omega) = \left(U_k^*(\omega)\hat{P}^{(l)}(\omega)\right) V_k(\omega)$$

- Inverse Fourier transform of $\hat{P}_k^{(p)}(\omega)$ to obtain $P_k^{(p)}(t)$
- Back-propagation of $P_k^{(p)}(t)$ to achieve refocusing on one specific defect

# Stopping Criterion

- Refocusing at the defect during the backward propagation
- The refocusing time is not *a priori* known
- Stopping criteria based on the minimization of some norm of appropriate field quantities
  - Shannon entropy
  - Bounded variation function of a field variable
  - Mathematical energy
  - Total energy

$$\mathcal{E}(t) = \frac{1}{2}(A\sigma, \sigma) + \frac{1}{2}(\rho v, v)$$

- Definition of the discrete energy density
- Normalization by its maximal value
- Computation of its $\mathsf{L}^1$ norm

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# Numerical example

- **Geometry :** rectangular domain $L_x = 69.037$ mm and $L_y = 62.747$ mm
- **Mesh :** $400 \times 400$ grid with rectangular elements
- **Material :** steel with Lamé coefficients $\lambda = 96.95$ GPa and $\mu = 76.17$ GPa
- **Velocities :** pressure waves $c_p = 5689.9$ m/s and shear waves $c_s = 3145.2$ m/s
- Array of 21 equidistant **sensors** that act as **sources** as well
- **Two defects :** one at $(0.6L_x, 0.25L_y)$ of area 0.1 $mm^2$ and second $(0.6L_x, 0.75L_y)$ of area 0.4 $mm^2$
- **Damage** is considered in the material by the degradation of both Lamé coefficients by 10 %
- **Excitation function :** Ricker pulse with central frequency 1 MHz

# Results

- Forward step and construction of the IRM
- Back-propagation of the field recorded when the source is on the central array element. Response time-history Refocusing
- SVD of the IRM
- Back-propagation of the field recorded when the source is on the central array element after projection on the singular vector corresponding to the
  - first singular value Response time-history Refocusing
  - second singular value Response time-history Refocusing
  - third singular value Response time-history Refocusing

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# Summary and Conclusions

- Time reversal for damage localization in elastic bounded media
- Application of the DORT method for selective refocusing
- Choice of an effective stopping criterion (Total energy)
- Absence of notable differences between array and distributed sensor configurations
- Difficulties in the elastic medium due to the two types of waves (pressure and shear) and their conversions
- Difficulties due to the presence of boundaries

# Future work

- Extensive investigation of the distributed sensor configuration
- Propose optimal total experiment time
- Application of imaging techniques [8]
- Investigation of the methodology using passive noisy recordings as input data
- Account for dissipation (damping) and dispersion
- Application to structures with complex geometry

8. L Borcea, G Papanicolaou, C Tsogka and J Berryman, (2002) Imaging and time reversal in random media

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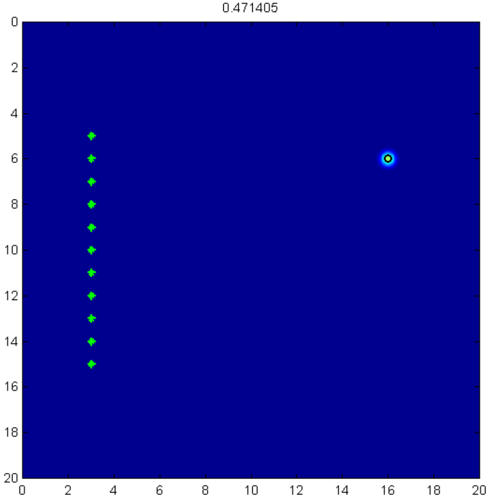


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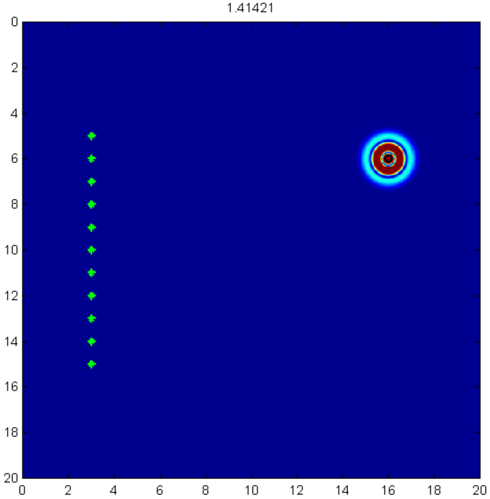

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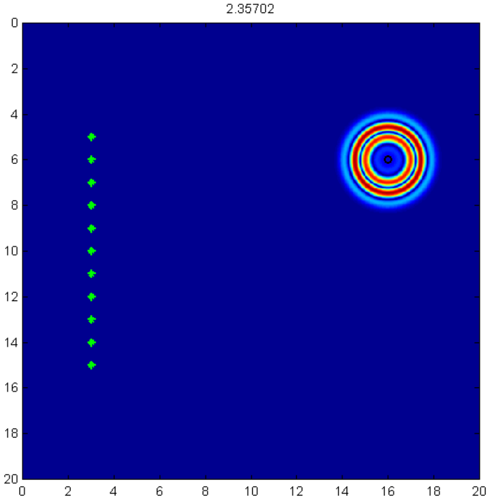

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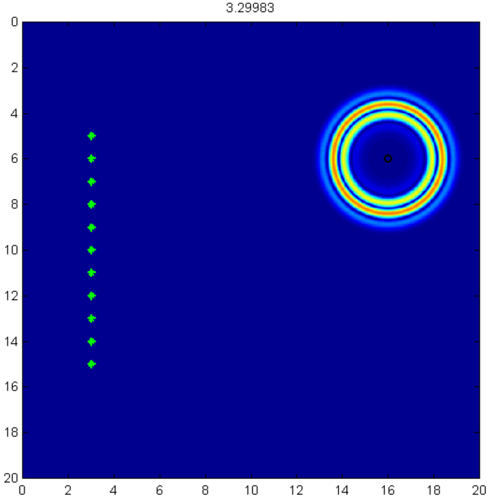


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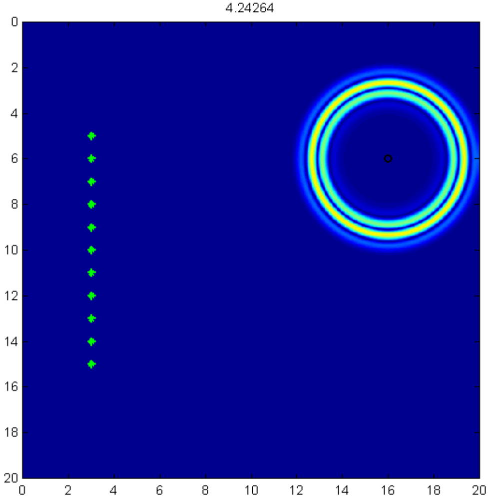


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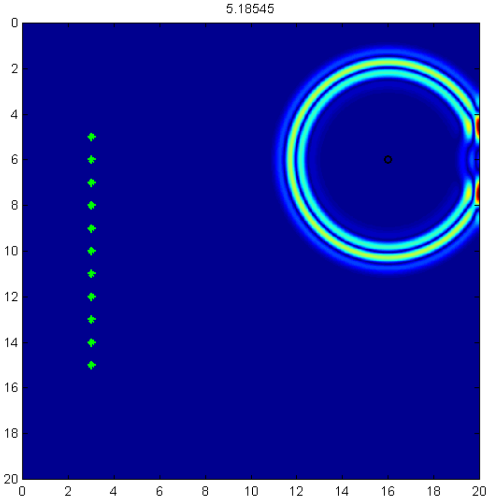

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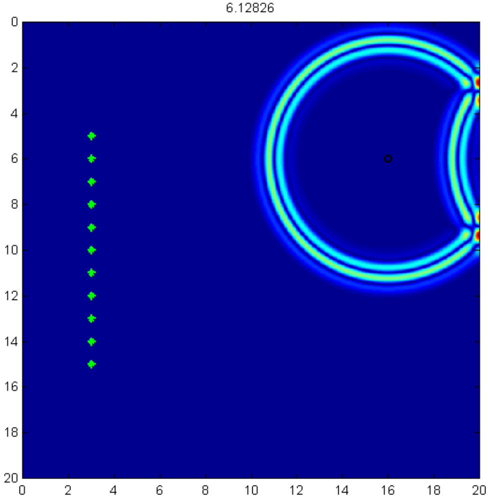


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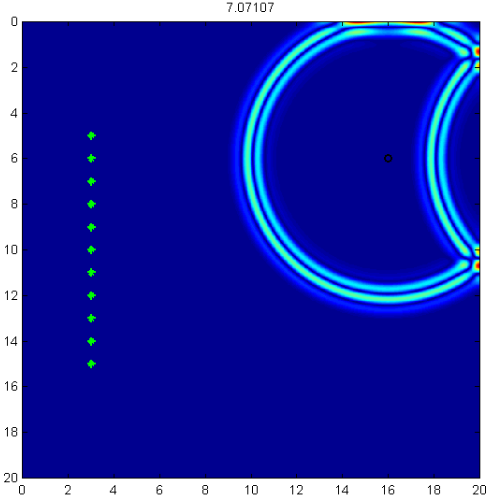


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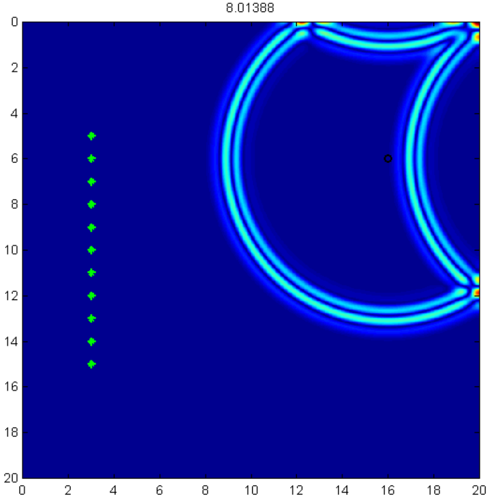


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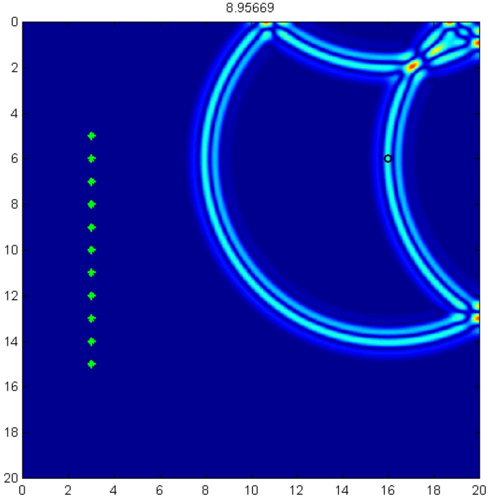


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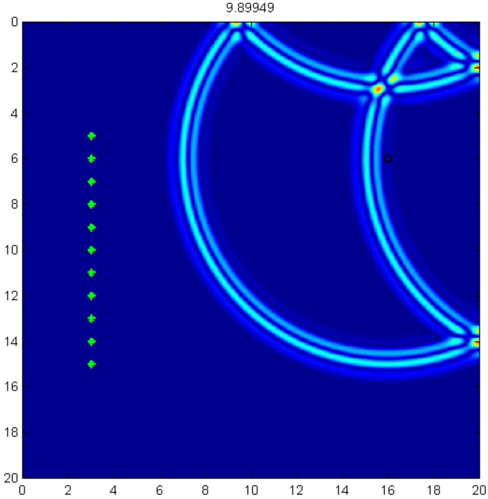


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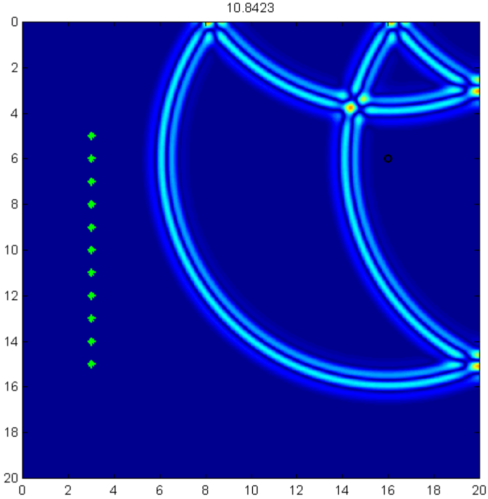


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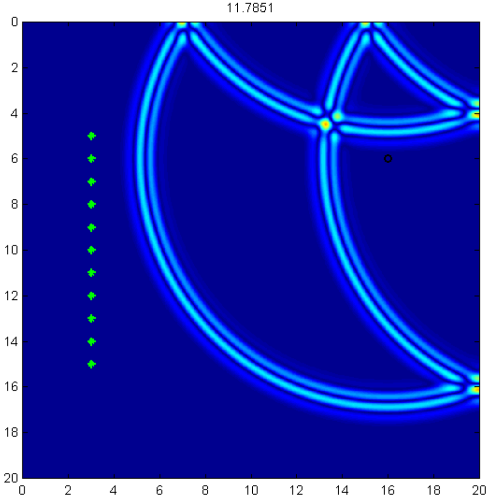


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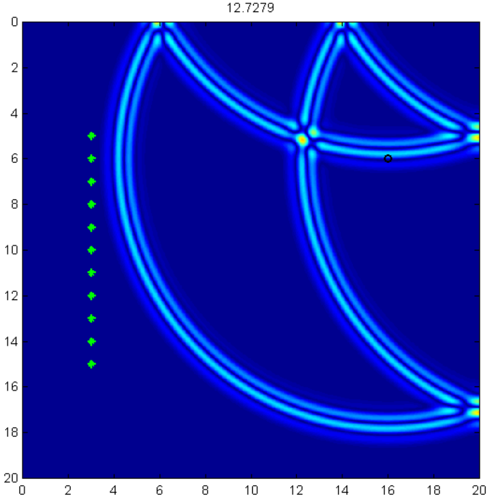

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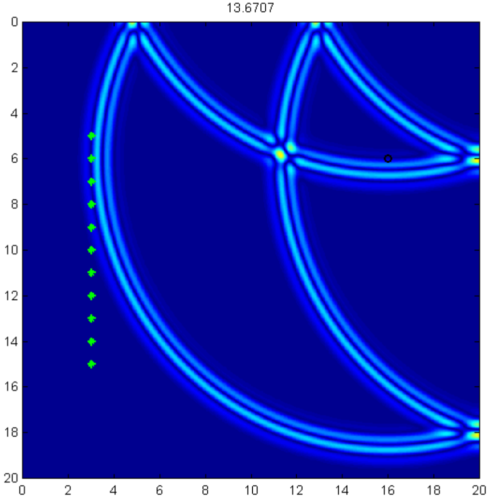


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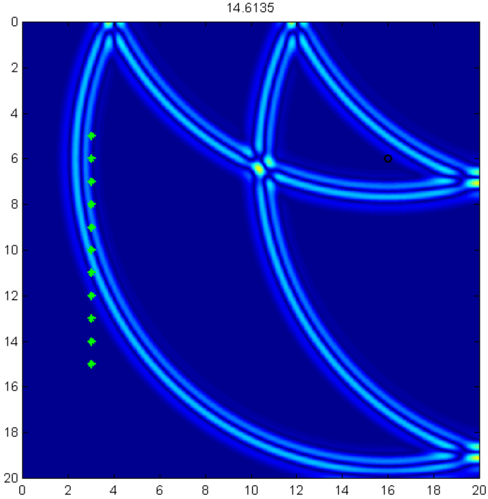

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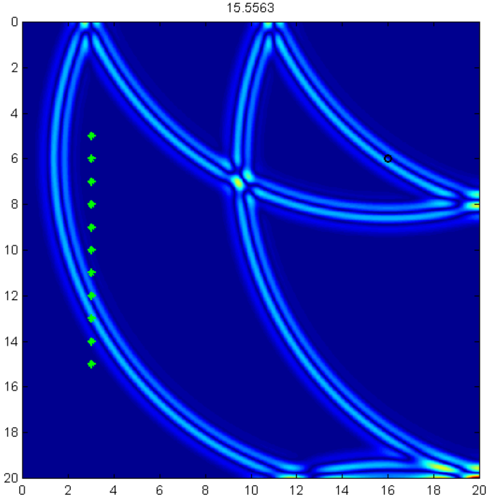


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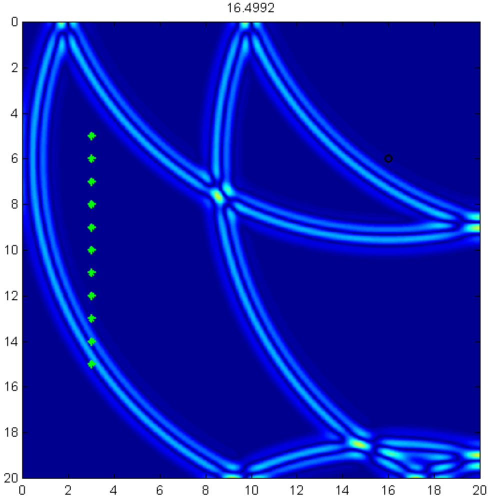

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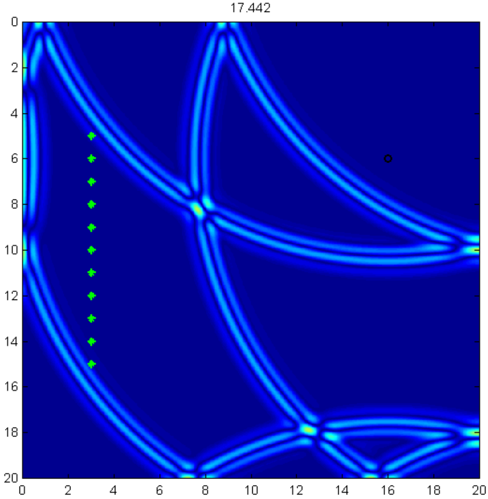


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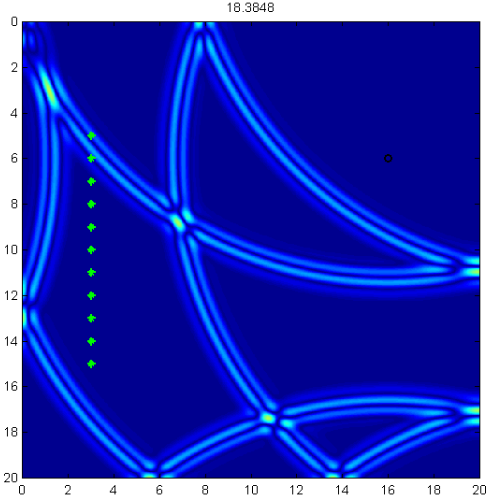


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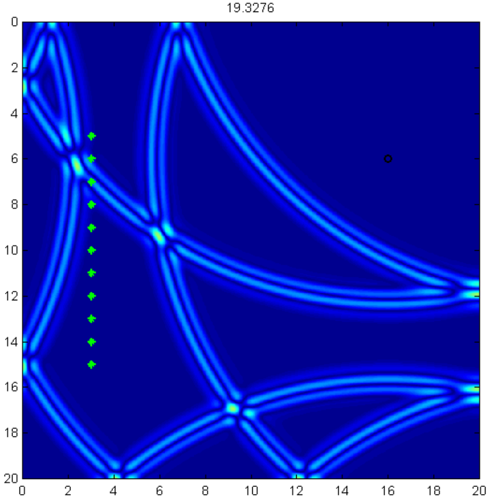


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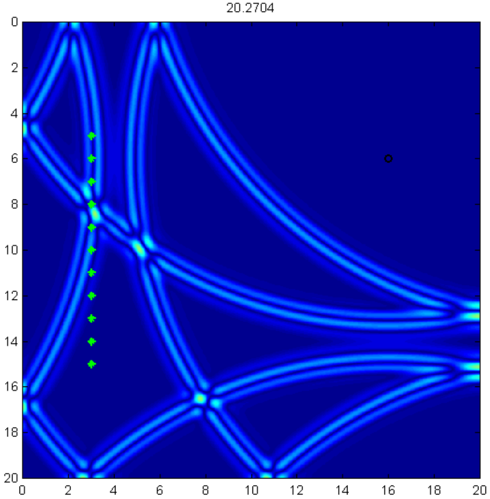


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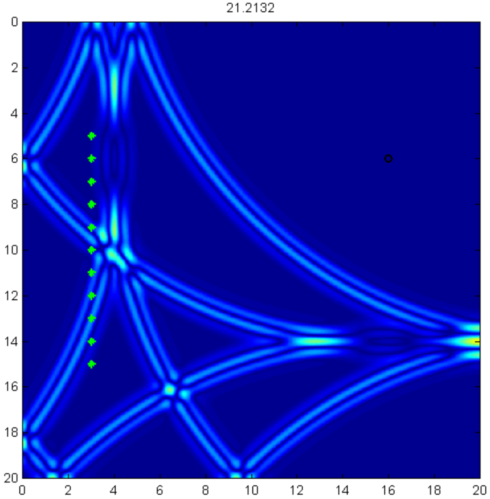


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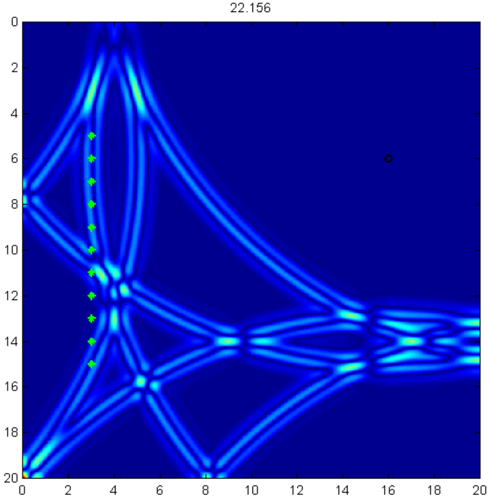


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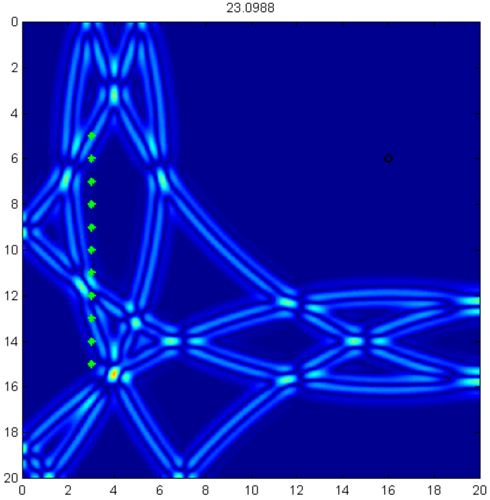
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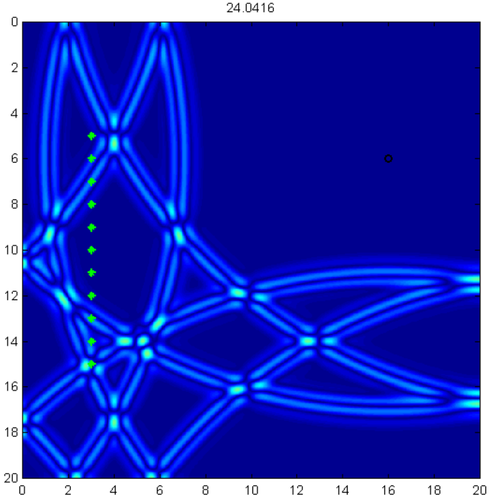


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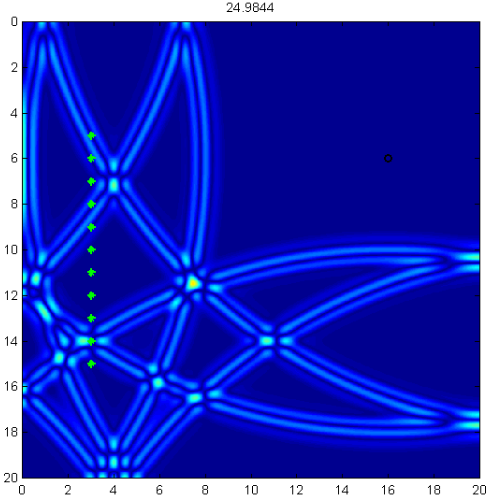


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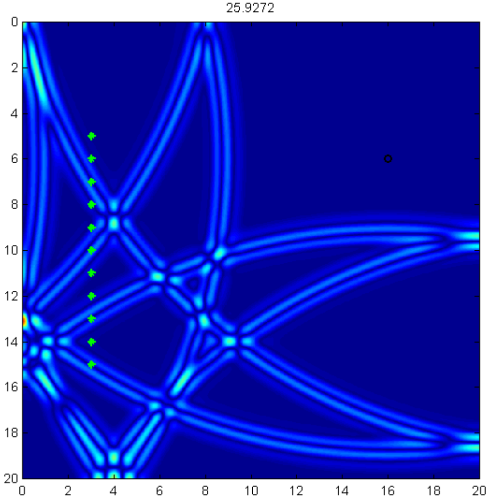


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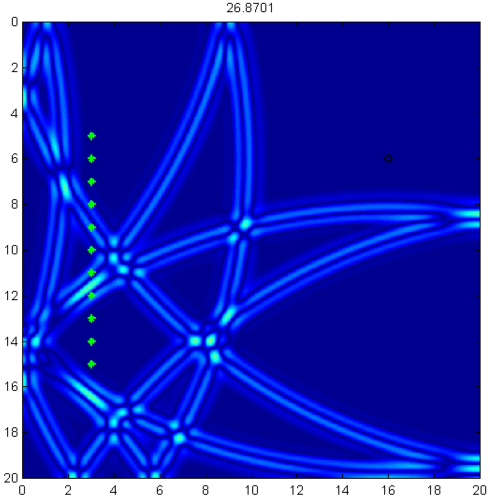


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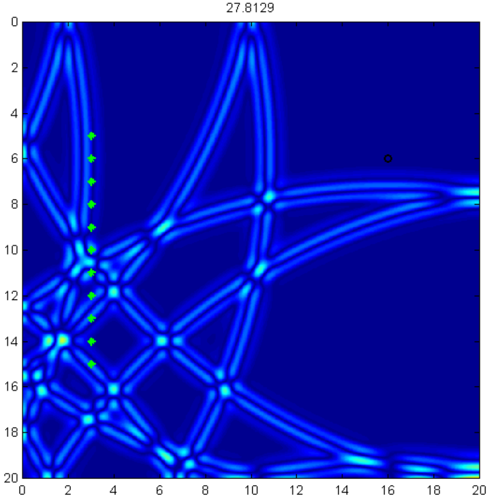


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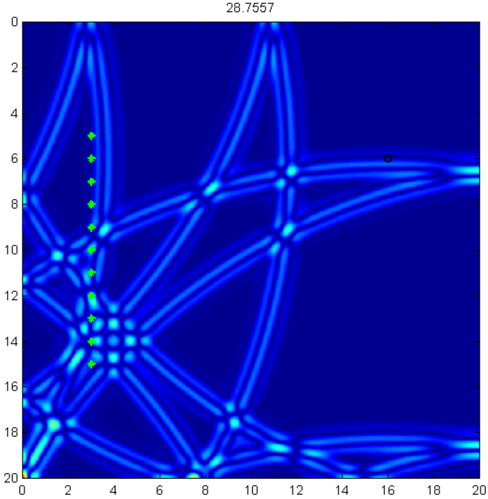

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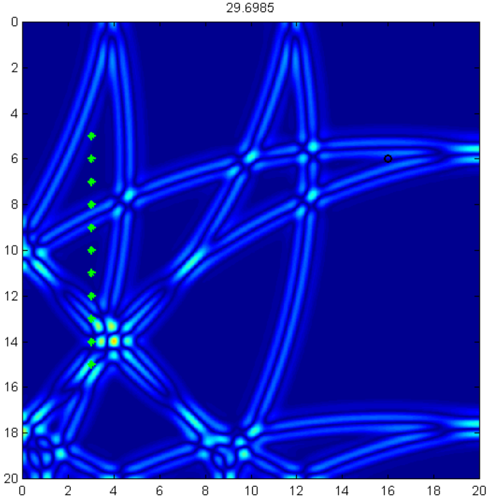


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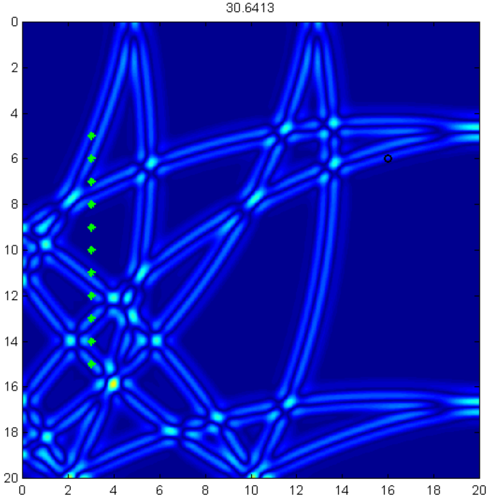


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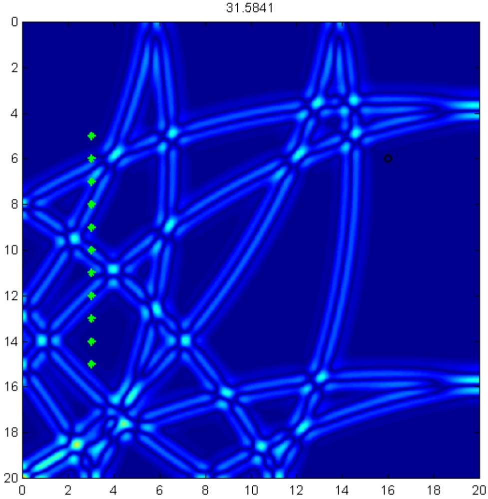

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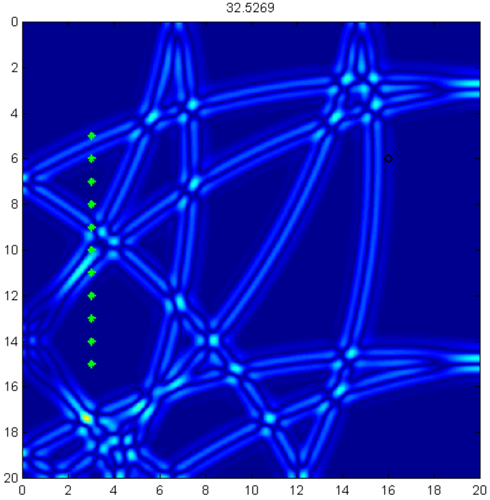


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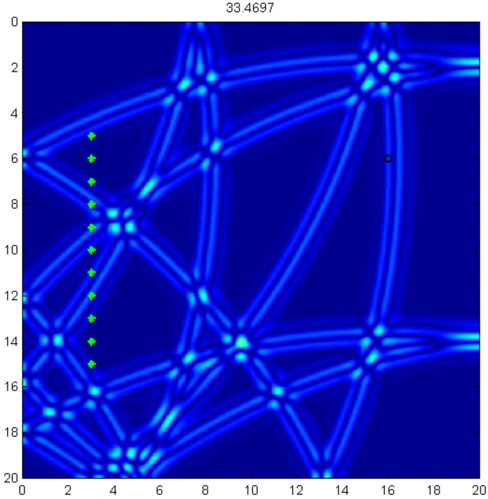


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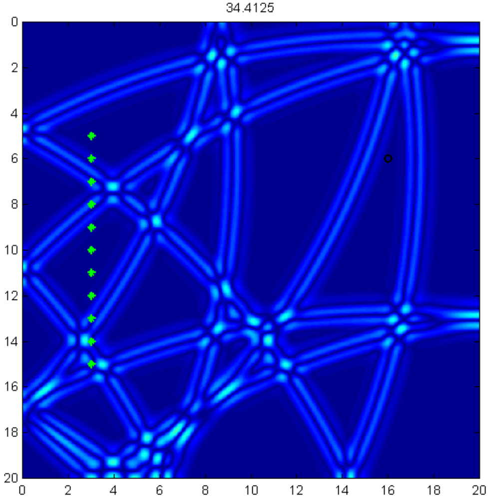


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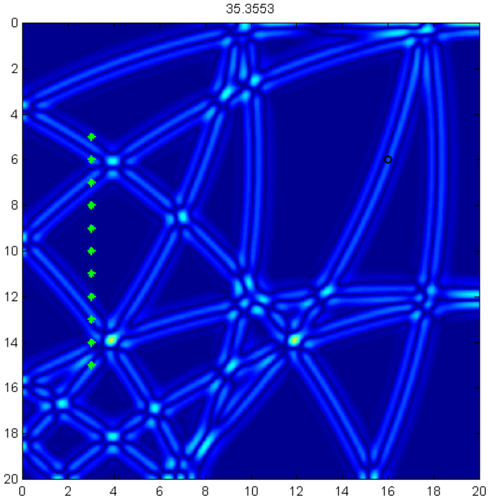


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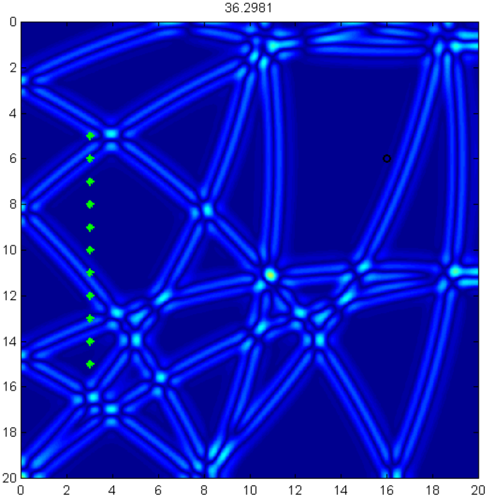


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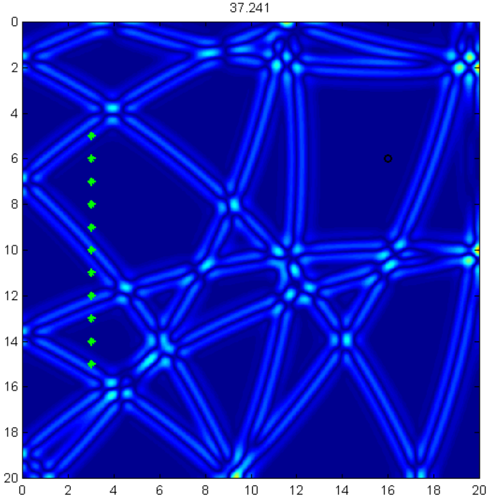


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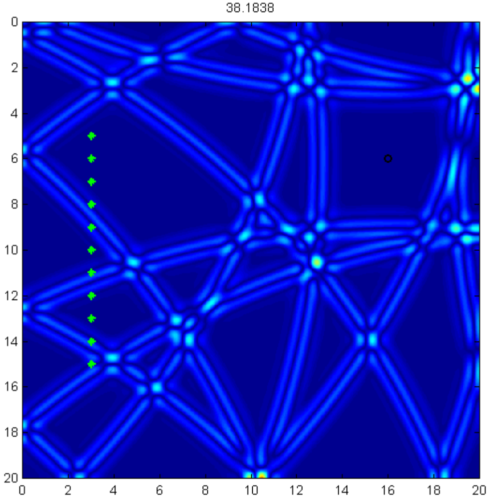


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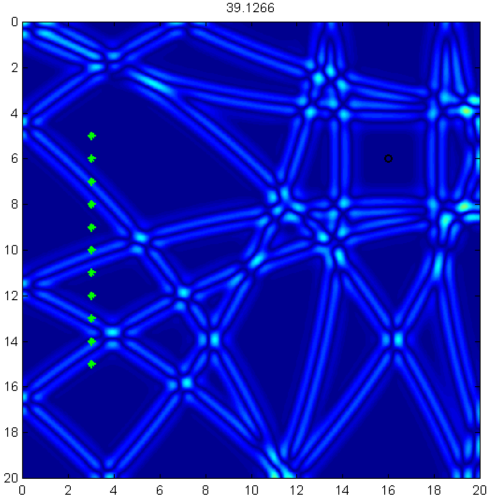


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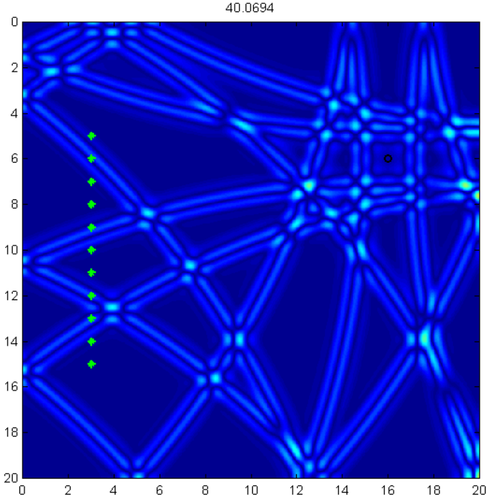


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- Time reverse the recordings and rebroadcast here

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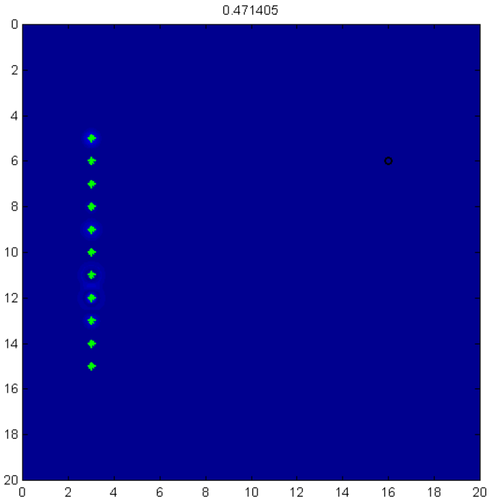


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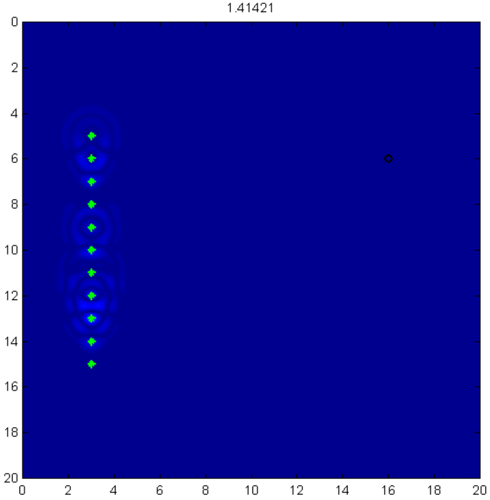

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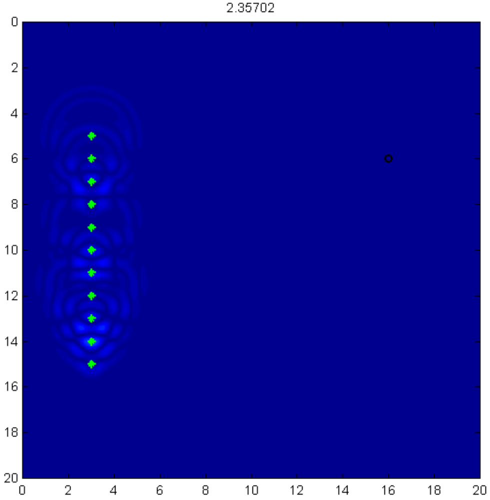


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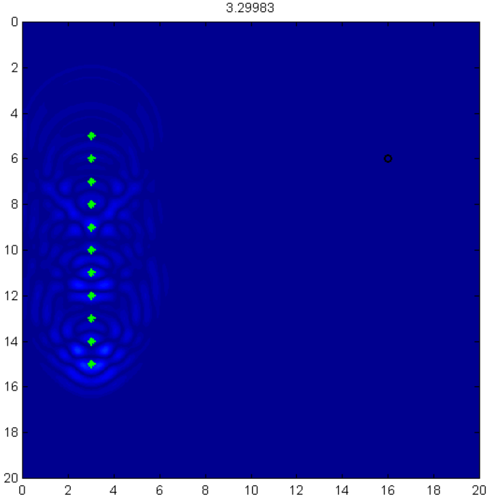


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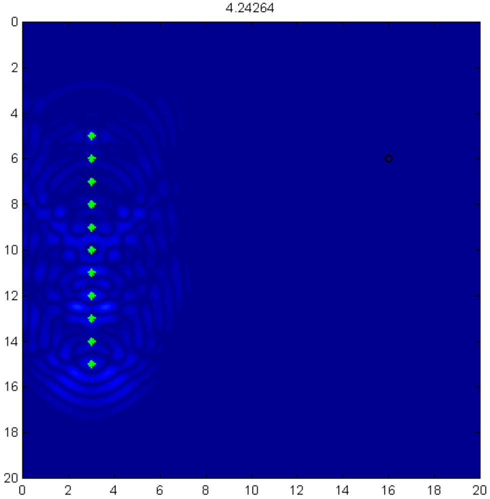


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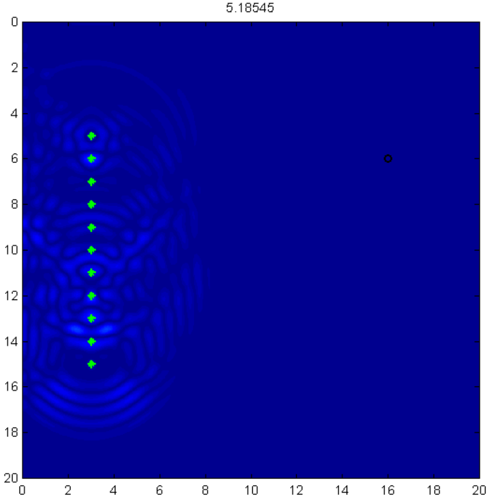


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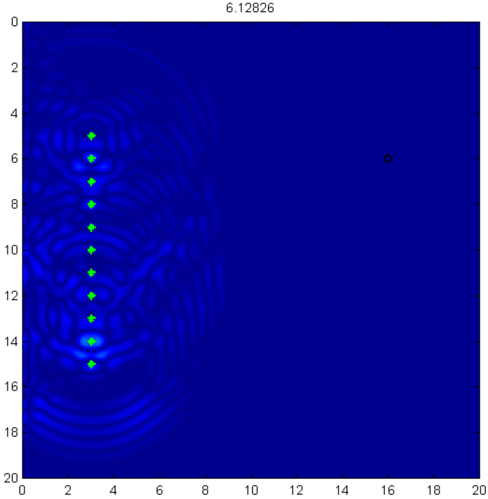
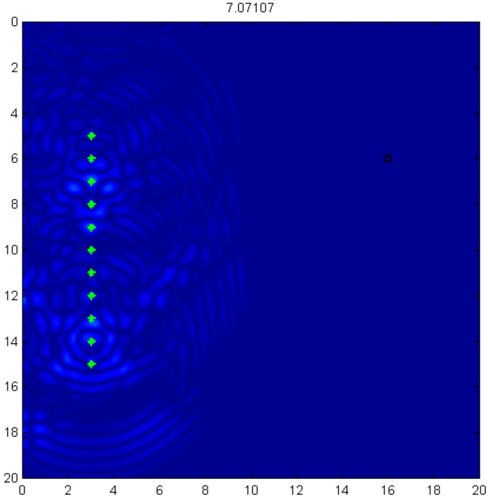
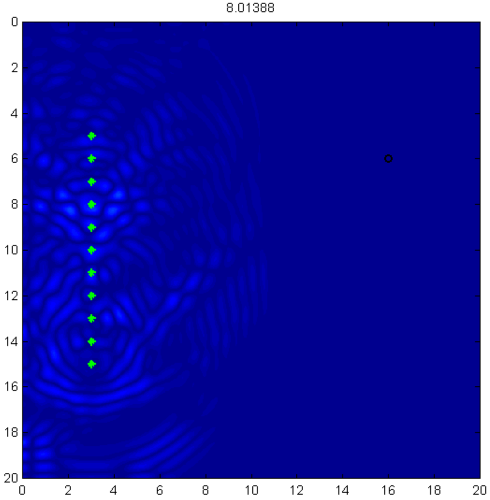



main

# Source localization

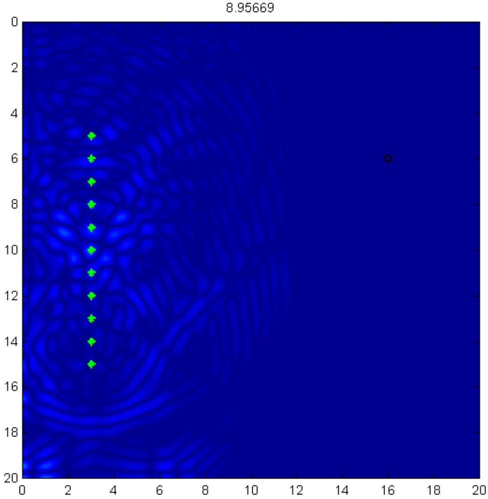


# Source localization




main

# Source localization




main

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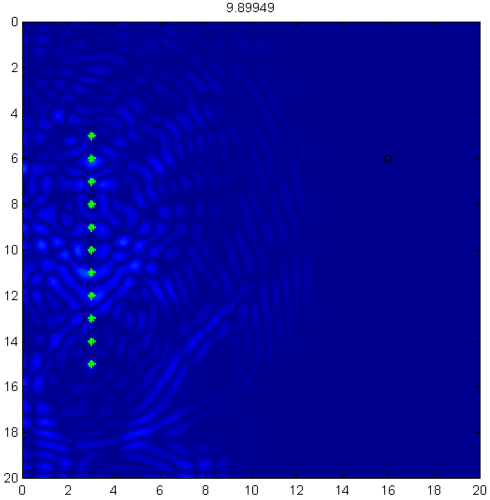
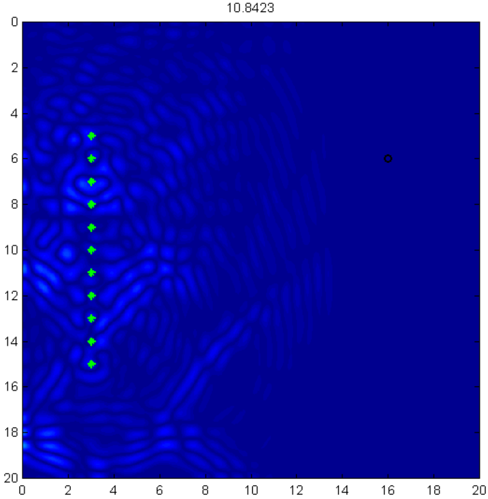
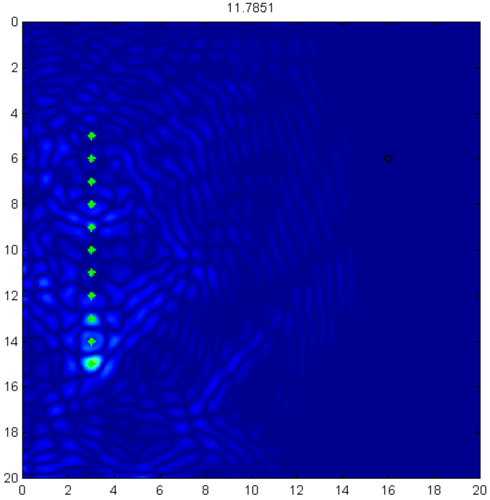



main

# Source localization



# Source localization




main

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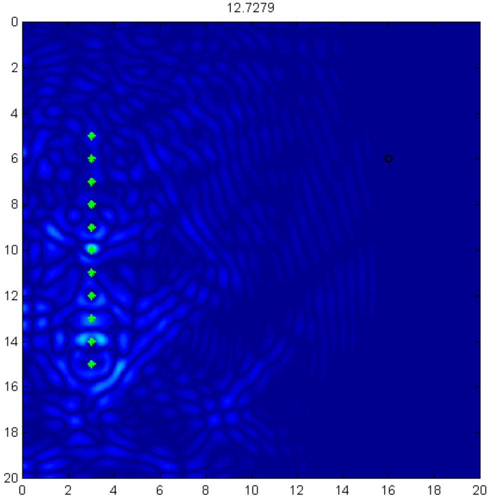
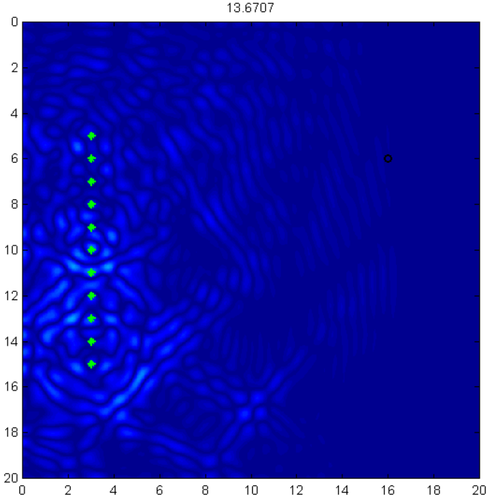
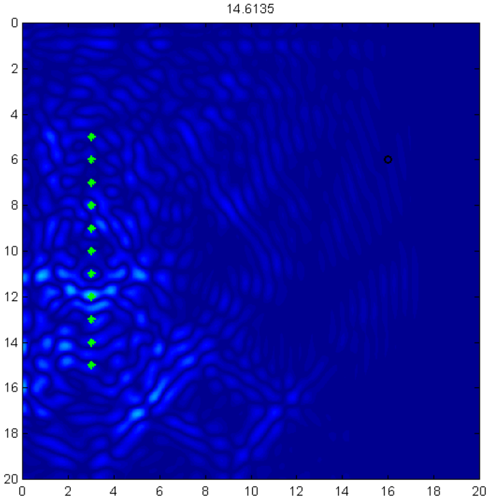


# Source localization



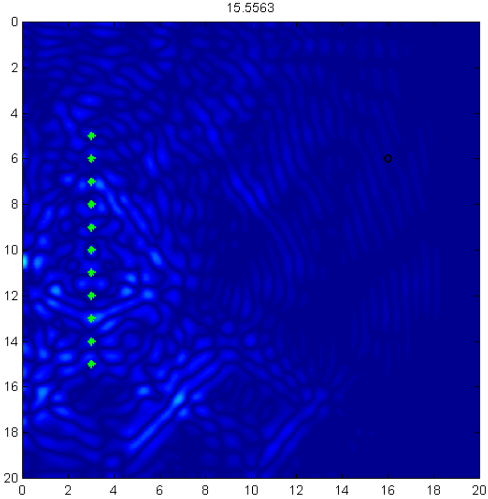

main

# Source localization



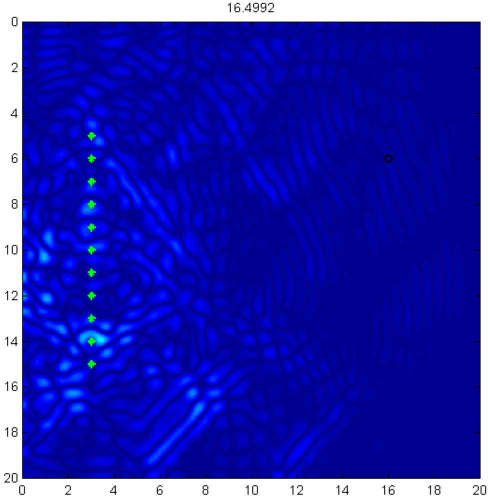

main

# Source localization

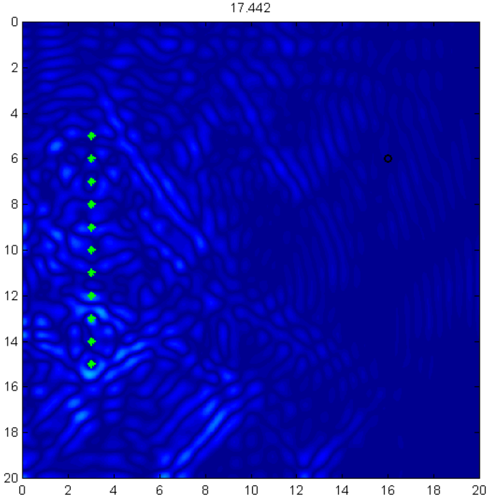



main

# Source localization



# Source localization




main

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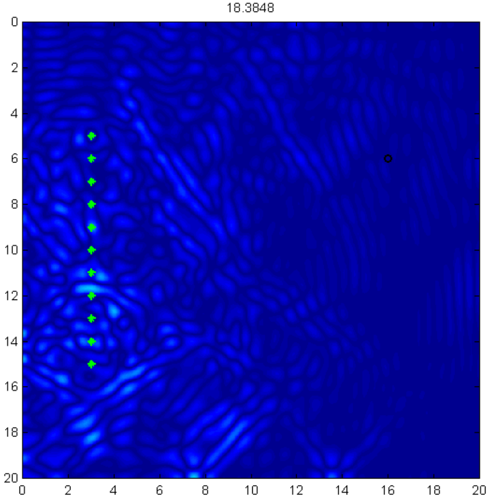
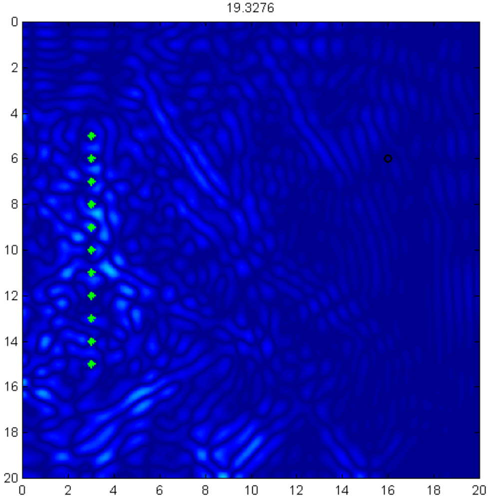


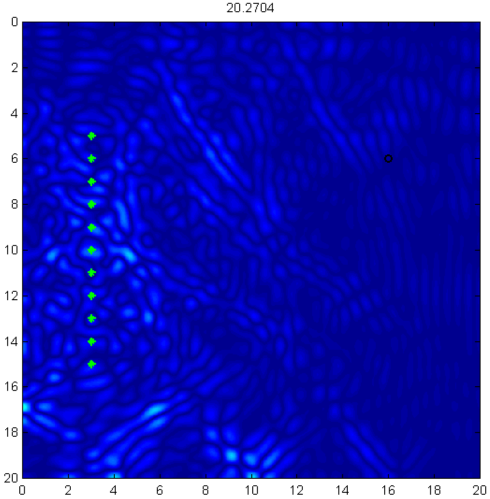

main

# Source localization



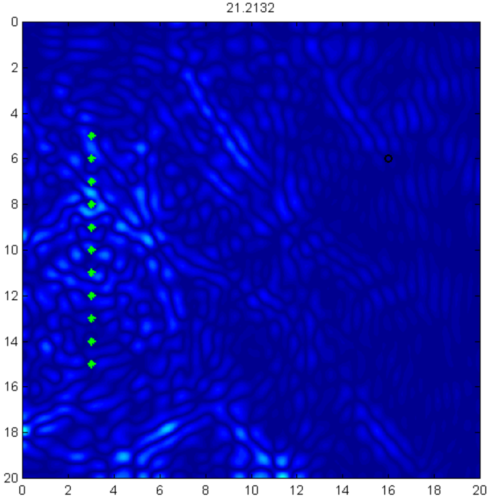

main

# Source localization




main

# Source localization

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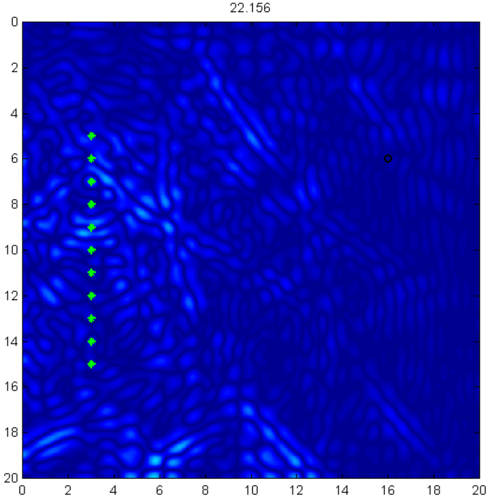
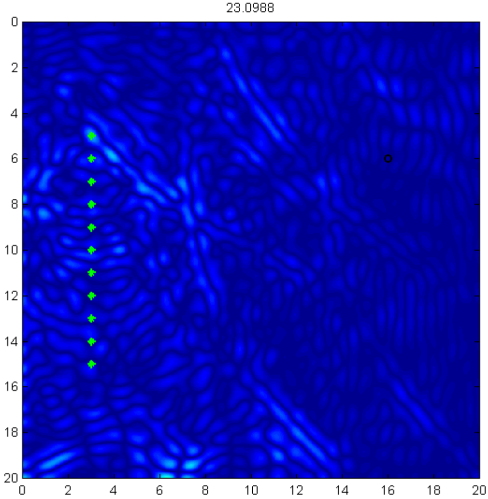


# Source localization

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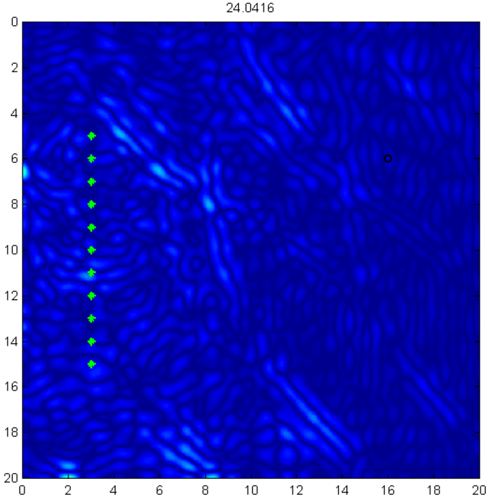
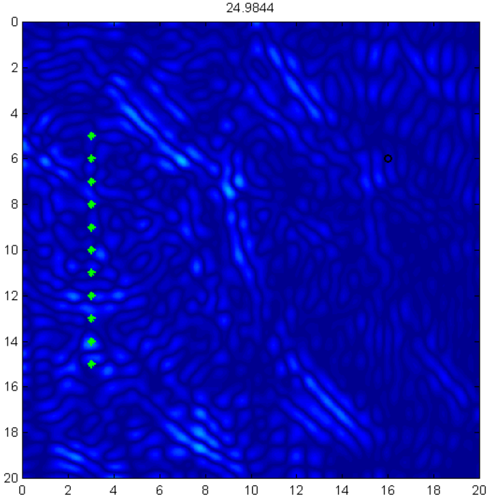
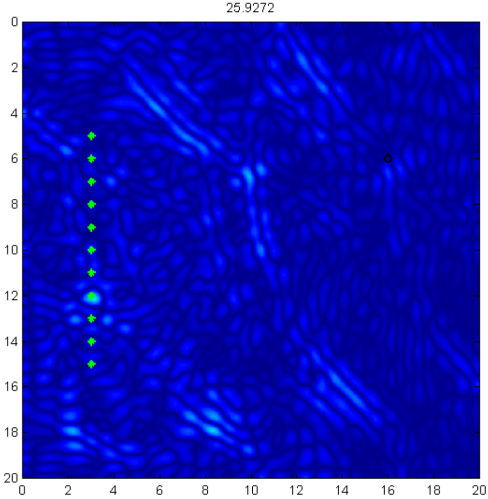



main

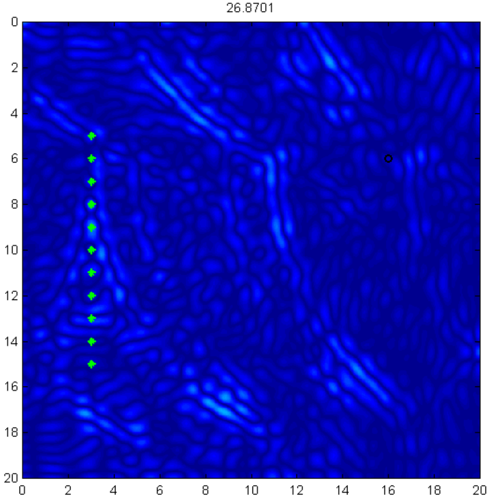
# Source localization



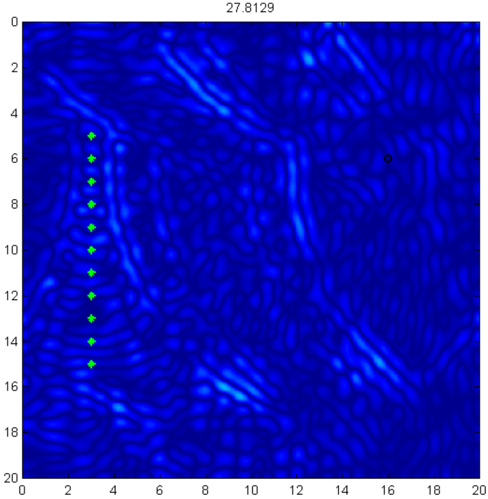
# Source localization



# Source localization

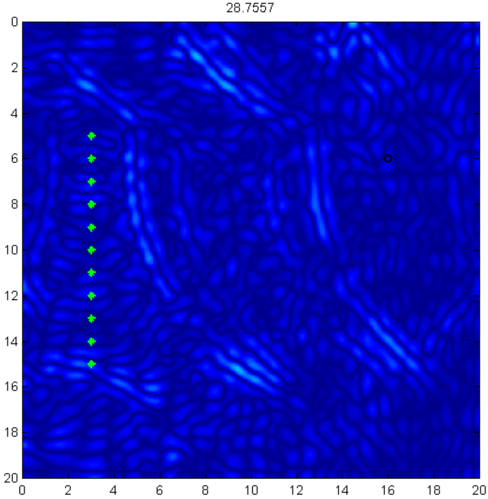


# Source localization

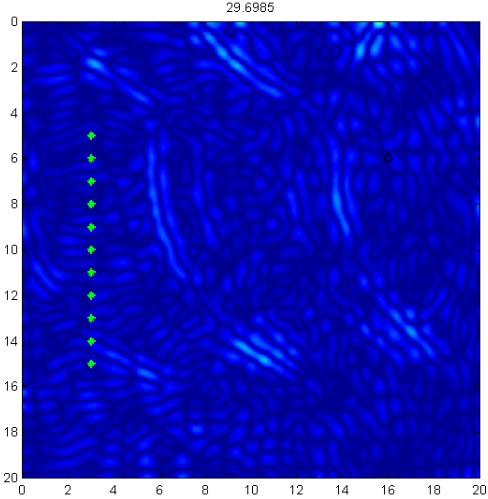



main

# Source localization



# Source localization




main

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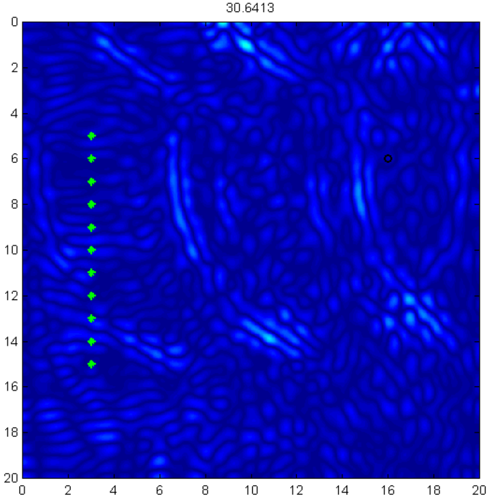


main

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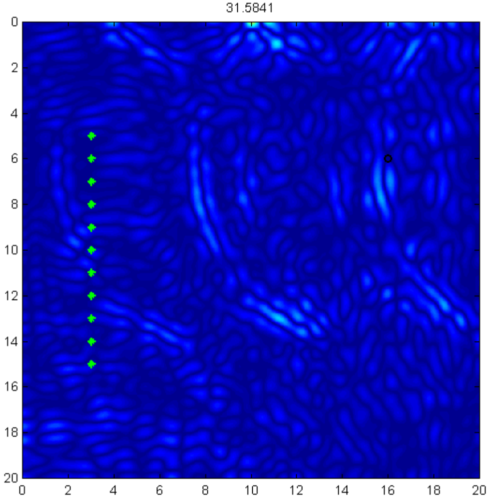
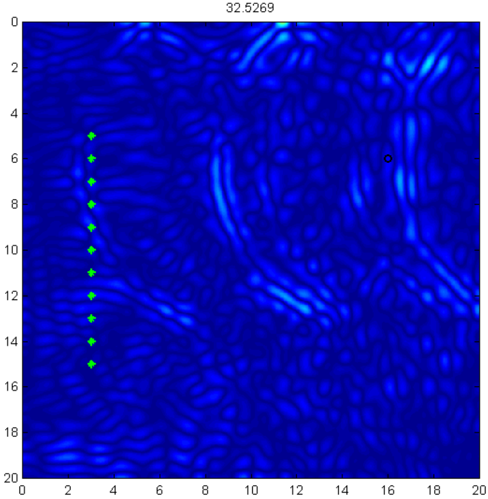
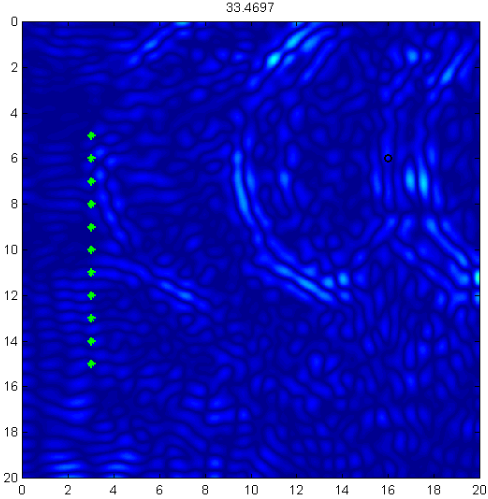



main

# Source localization

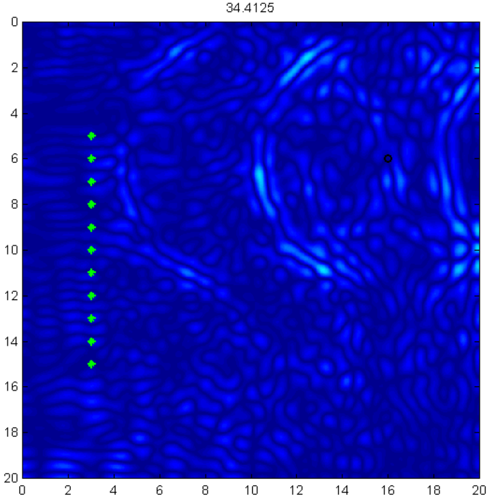


# Source localization

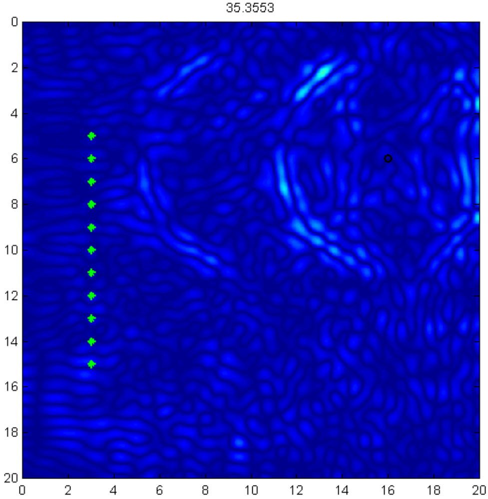



main

# Source localization



# Source localization




main

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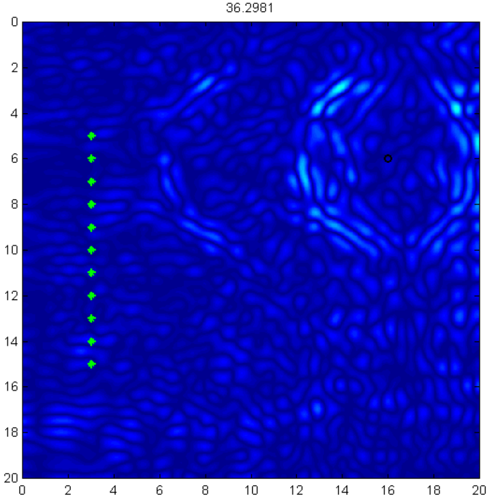


main

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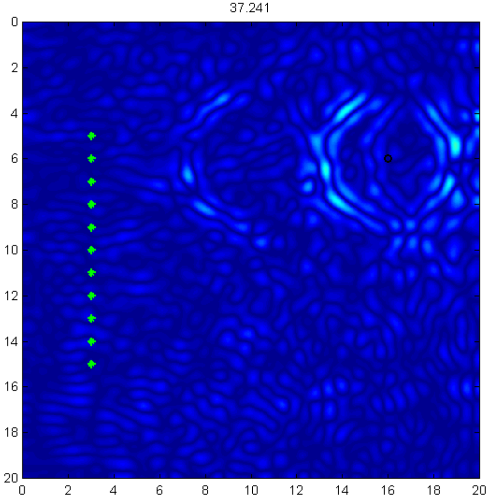


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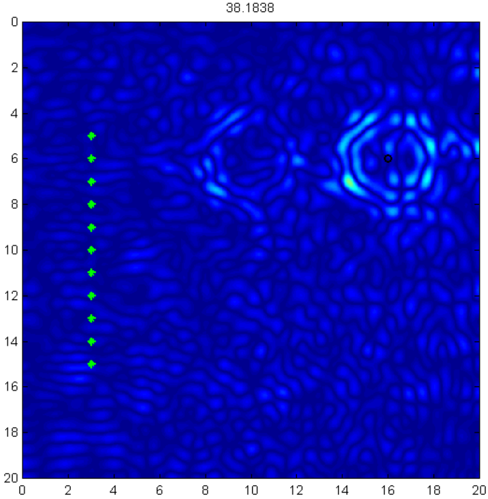
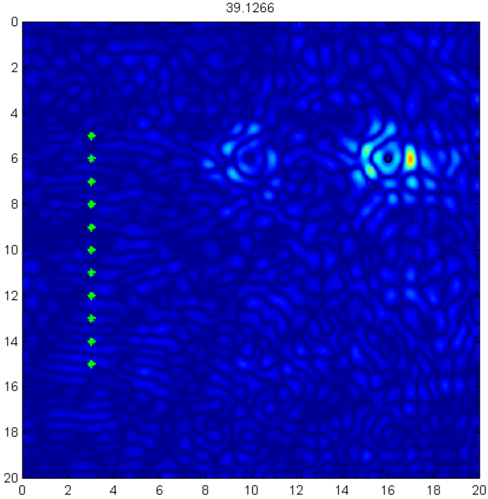
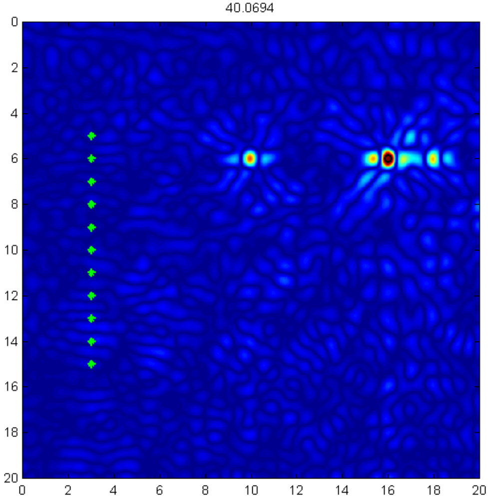



main

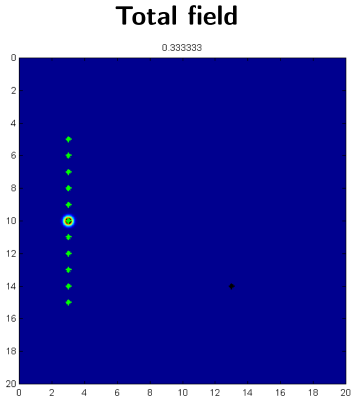
# Source localization

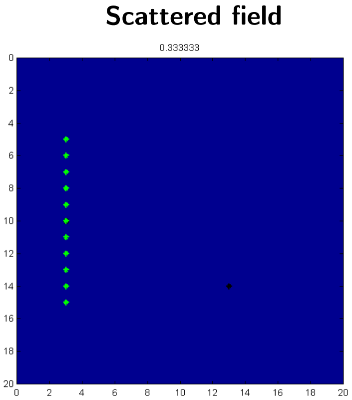


# Source localization



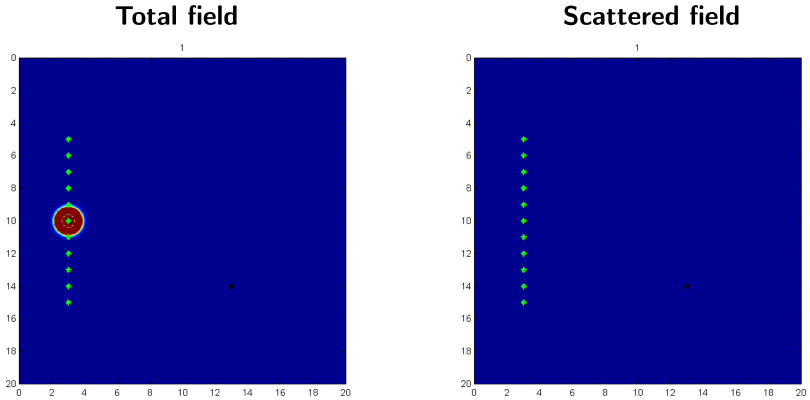
# Defect localization - forward step



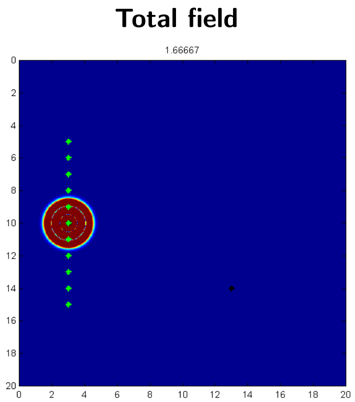

main

# Defect localization - forward step



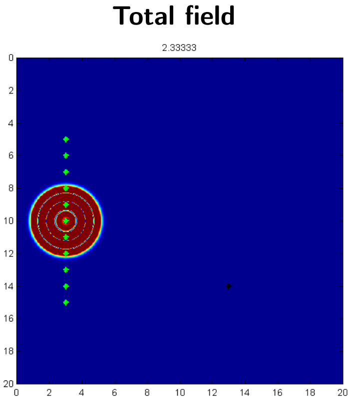

main

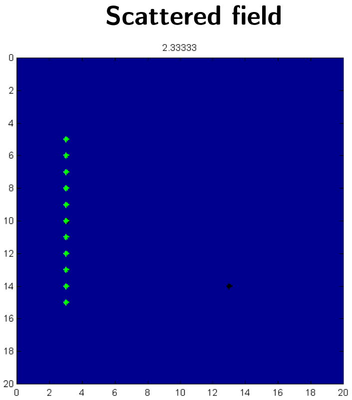
# Defect localization - forward step




main

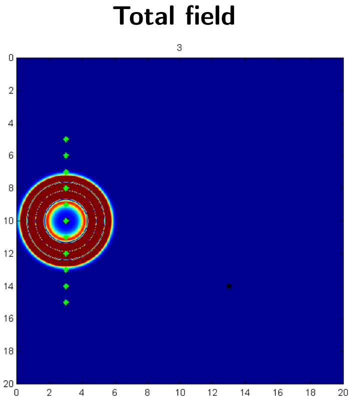
# Defect localization - forward step

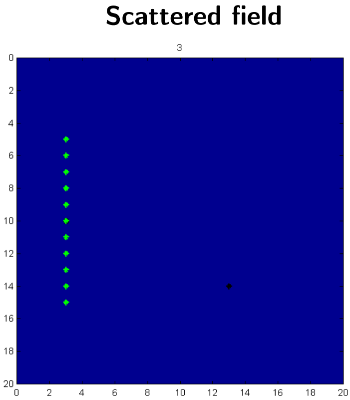



main

# Defect localization - forward step

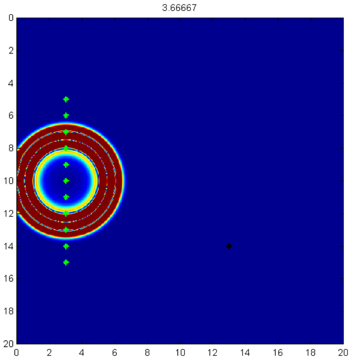


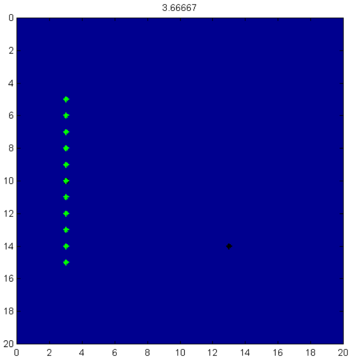

main

# Defect localization - forward step

**Total field**




**Scattered field**




main

# Defect localization - forward step

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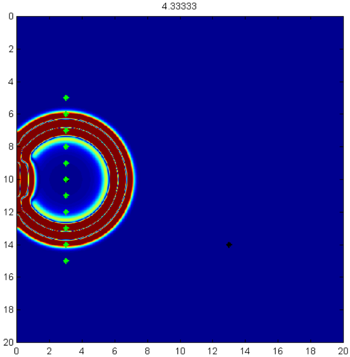
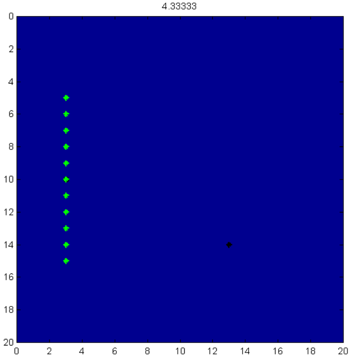


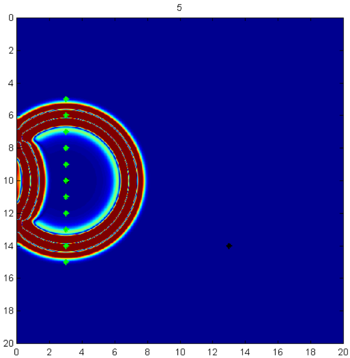

**Scattered field**



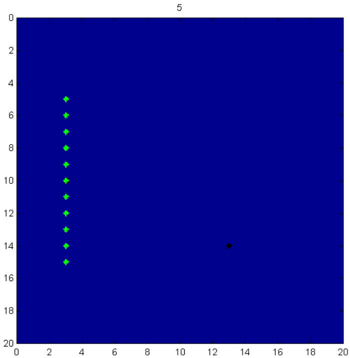

main

# Defect localization - forward step

**Total field**



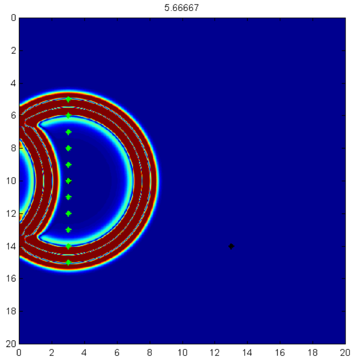

**Scattered field**



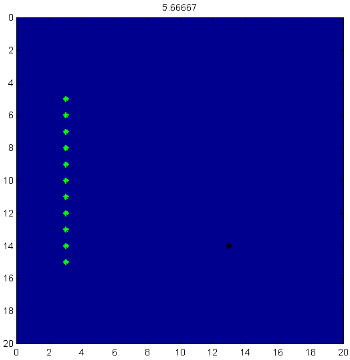

main

# Defect localization - forward step

## Total field




## Scattered field




main

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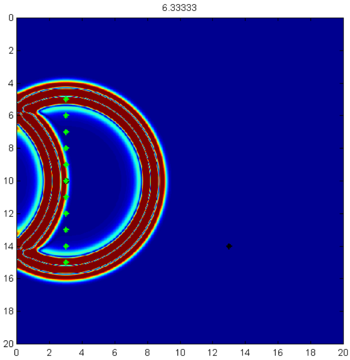
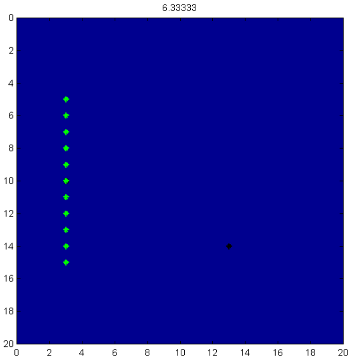
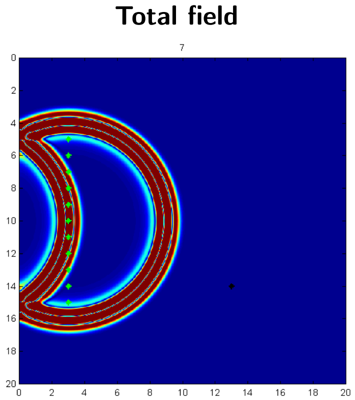


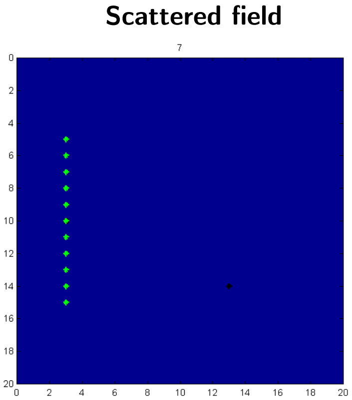

**Scattered field**




main

# Defect localization - forward step

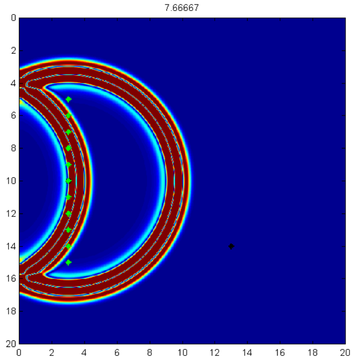


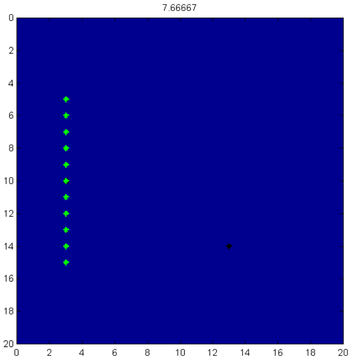

main

# Defect localization - forward step

**Total field**



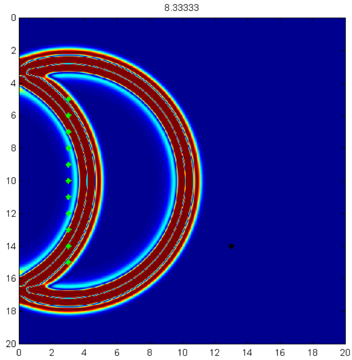

**Scattered field**



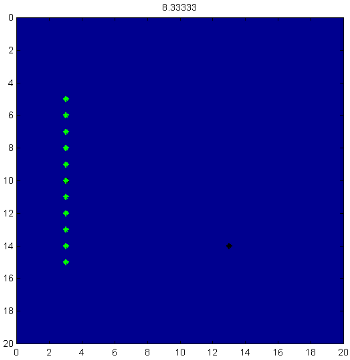

main

# Defect localization - forward step

**Total field**




**Scattered field**




main

# Defect localization - forward step

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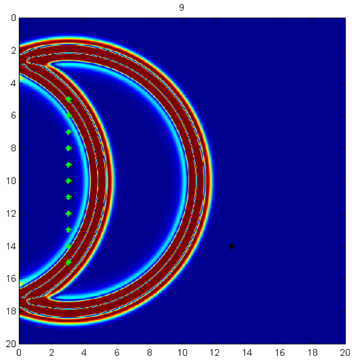
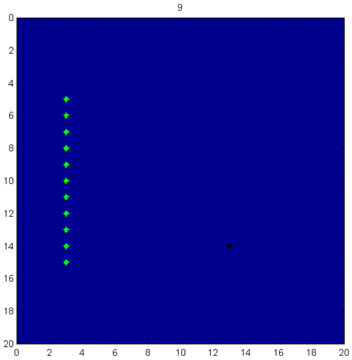



**Scattered field**




main

# Defect localization - forward step

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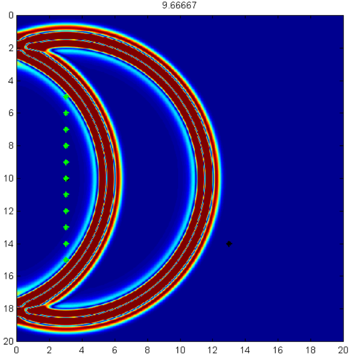
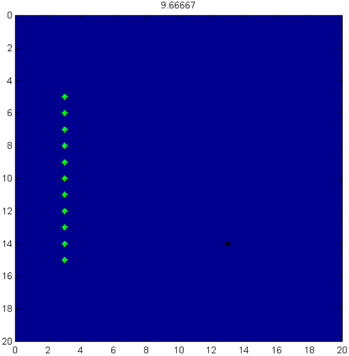


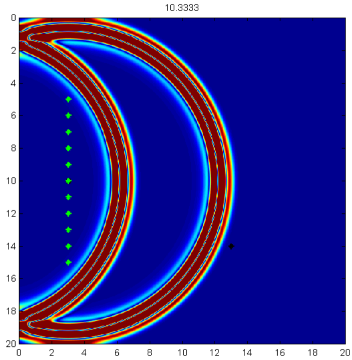

**Scattered field**



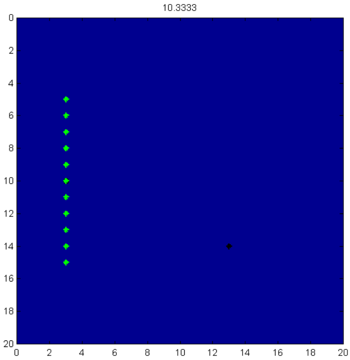

main

# Defect localization - forward step

**Total field**



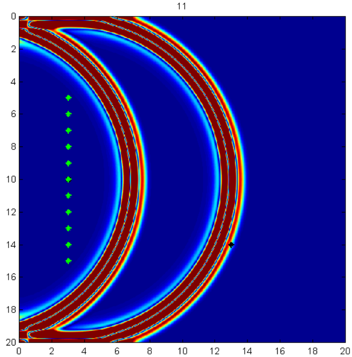

**Scattered field**



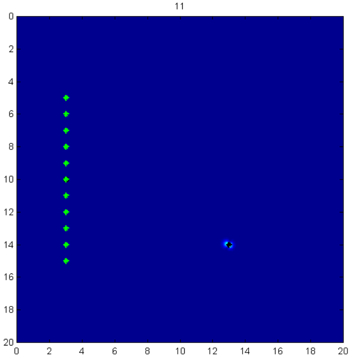

main

# Defect localization - forward step

**Total field**

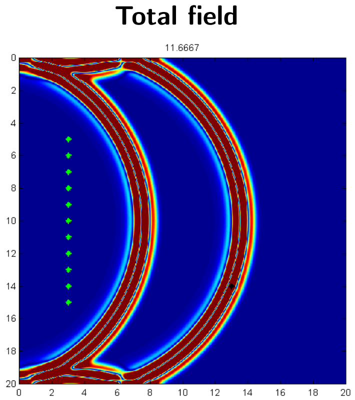


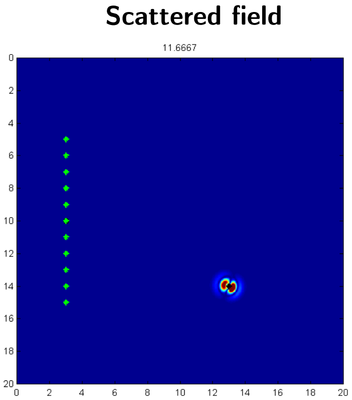

**Scattered field**




main

# Defect localization - forward step

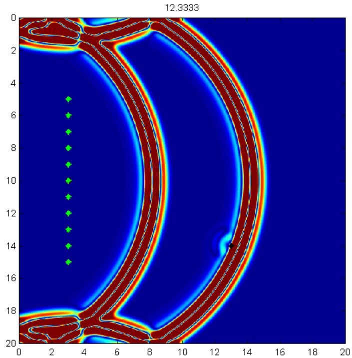


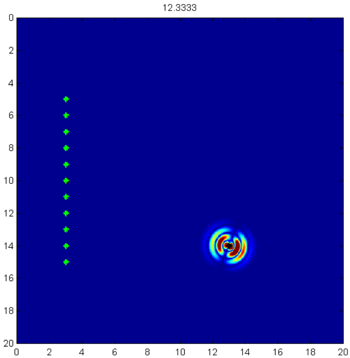

main

# Defect localization - forward step

## Total field



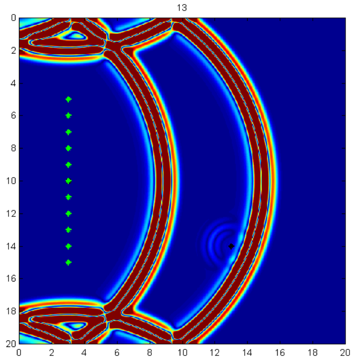

## Scattered field



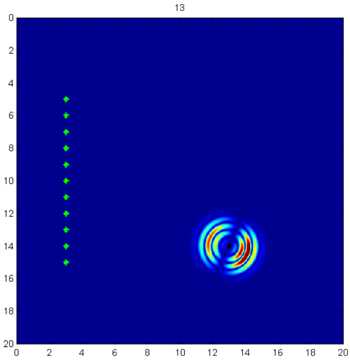

main

# Defect localization - forward step

**Total field**

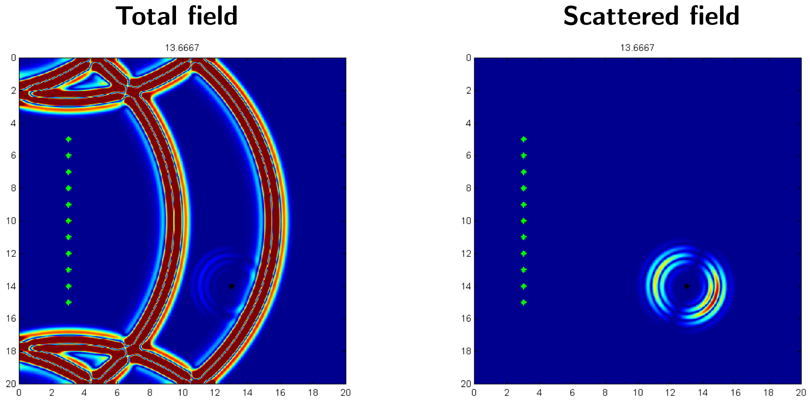



**Scattered field**




main

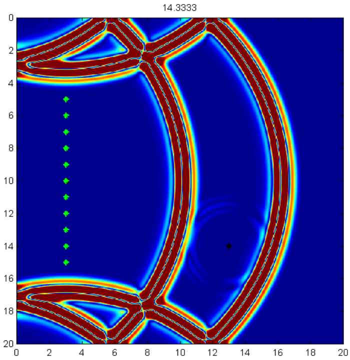
# Defect localization - forward step



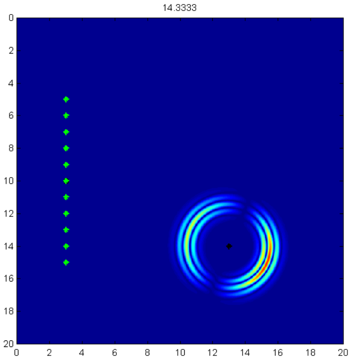

main

# Defect localization - forward step

**Total field**

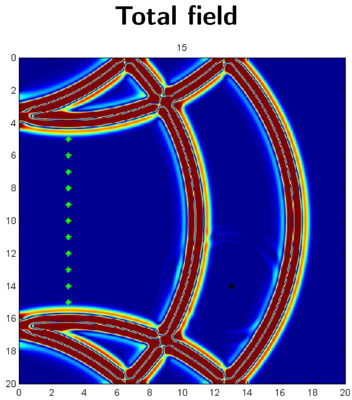


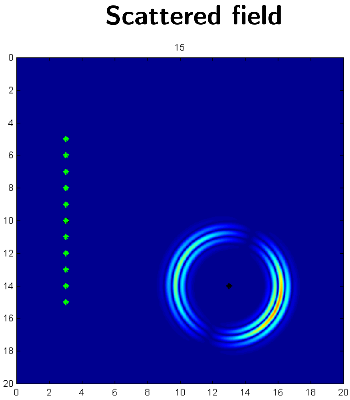

**Scattered field**




main

# Defect localization - forward step

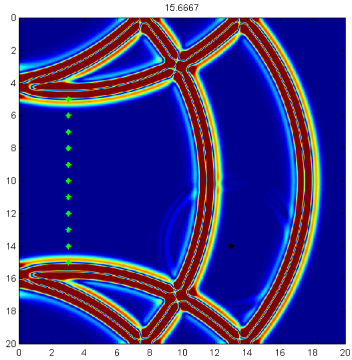


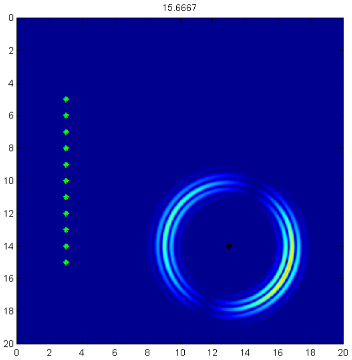

main

# Defect localization - forward step

**Total field**

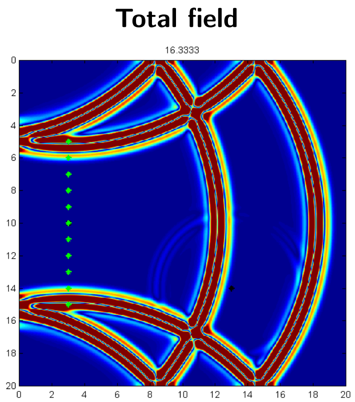


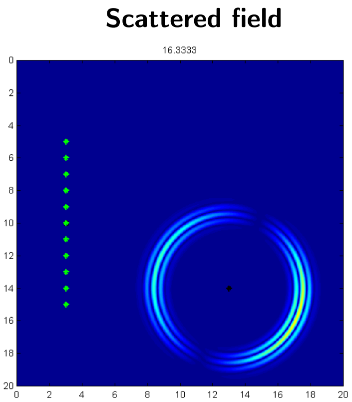

**Scattered field**




main

# Defect localization - forward step

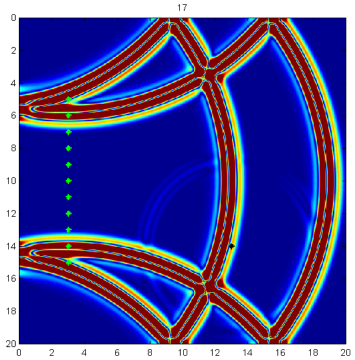


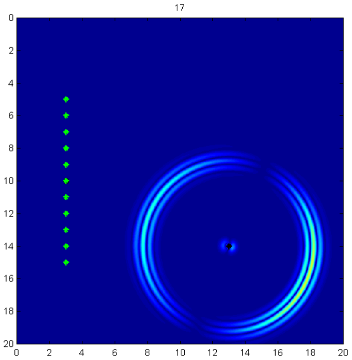

main

# Defect localization - forward step

**Total field**



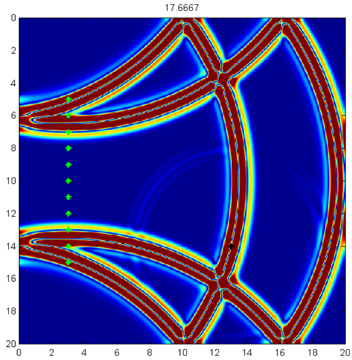

**Scattered field**



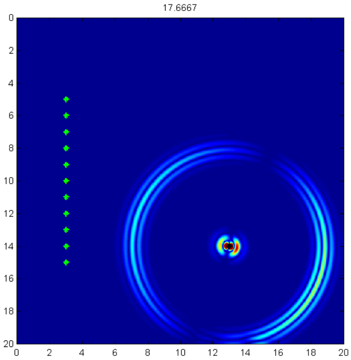

main

# Defect localization - forward step

**Total field**

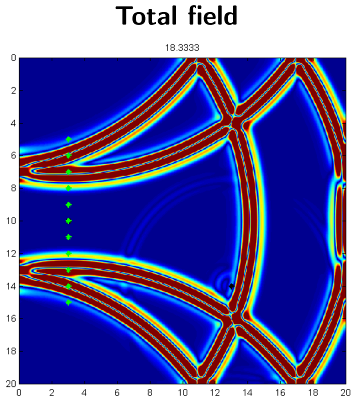


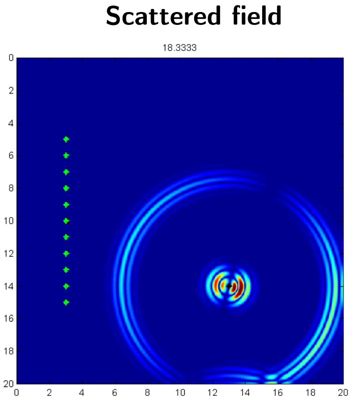

**Scattered field**




main

# Defect localization - forward step

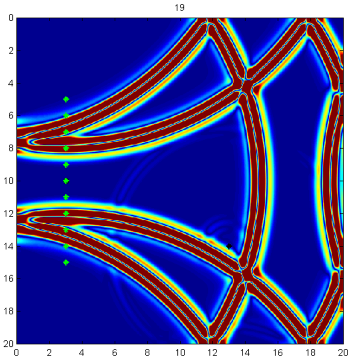


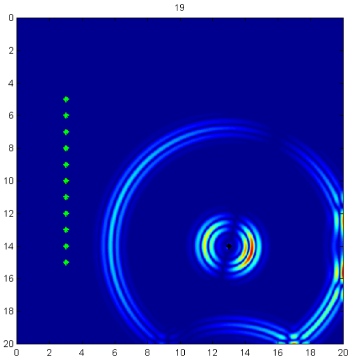

main

# Defect localization - forward step

**Total field**



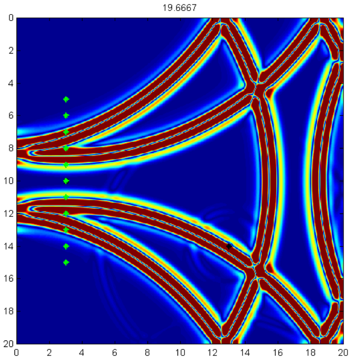

**Scattered field**



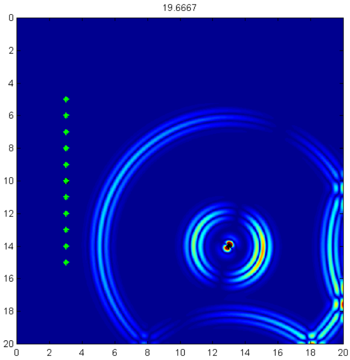

main

# Defect localization - forward step

**Total field**

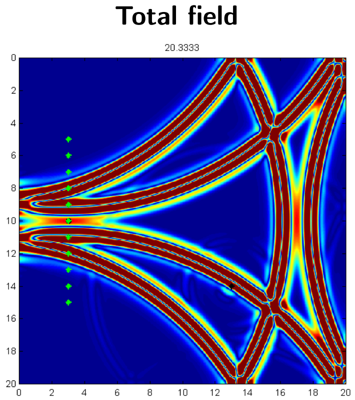


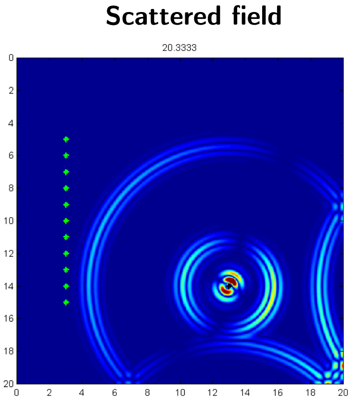

**Scattered field**




main

# Defect localization - forward step

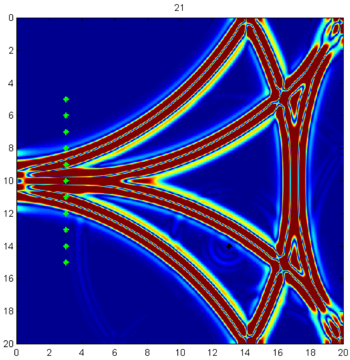


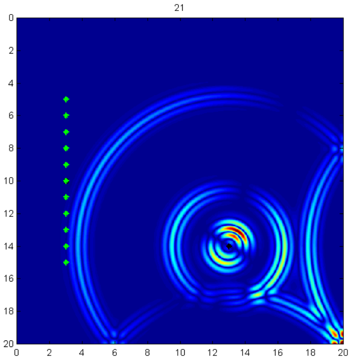

main

# Defect localization - forward step

**Total field**



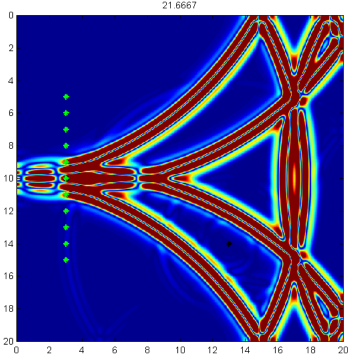

**Scattered field**




main

# Defect localization - forward step

**Total field**

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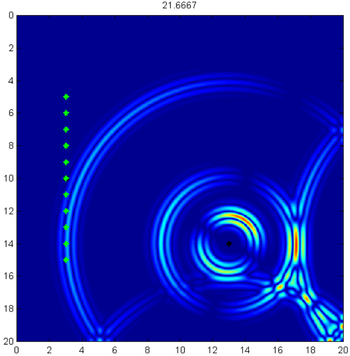
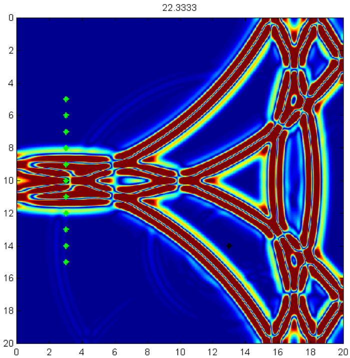


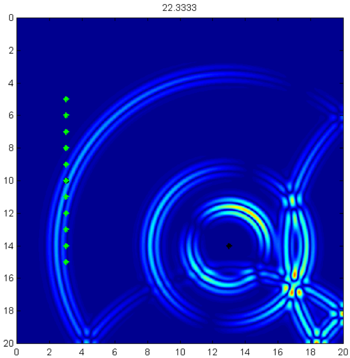

main

# Defect localization - forward step

**Total field**

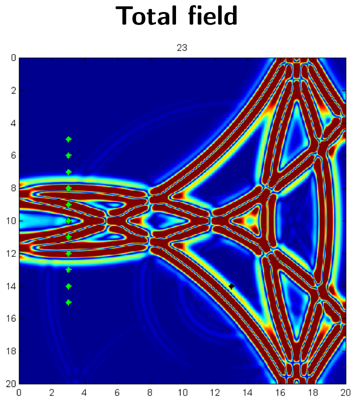


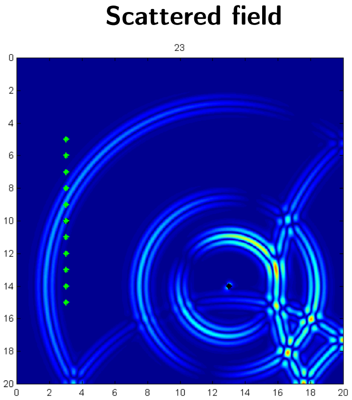

**Scattered field**




main

# Defect localization - forward step

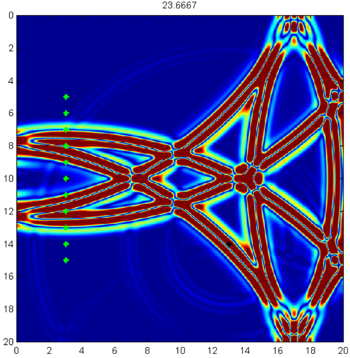


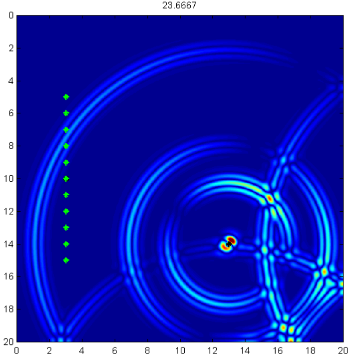

main

# Defect localization - forward step

**Total field**



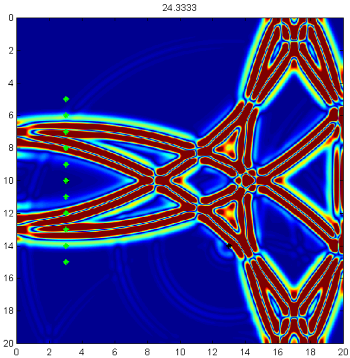

**Scattered field**



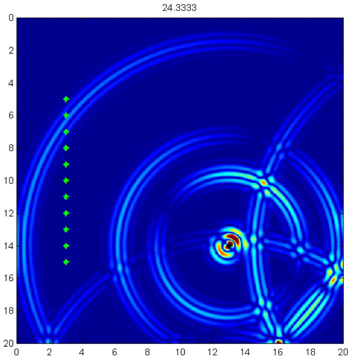

main

# Defect localization - forward step

**Total field**




**Scattered field**




main

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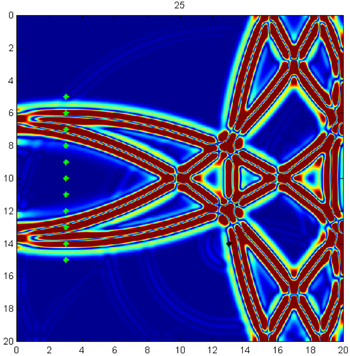


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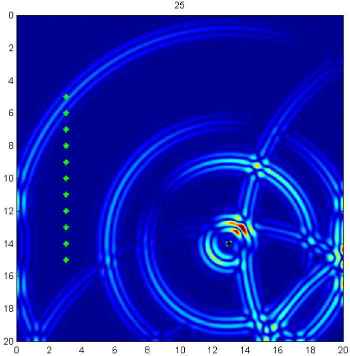
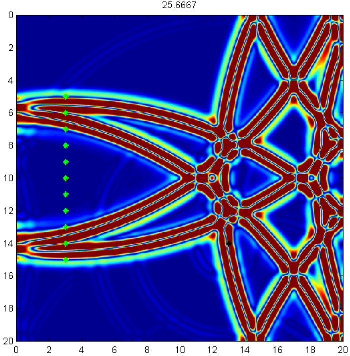



main

# Defect localization - forward step

**Total field**

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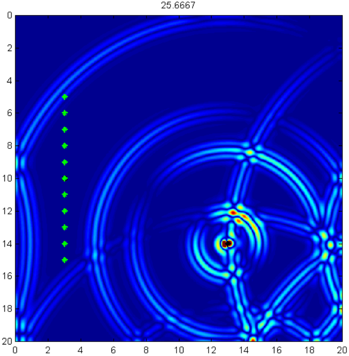
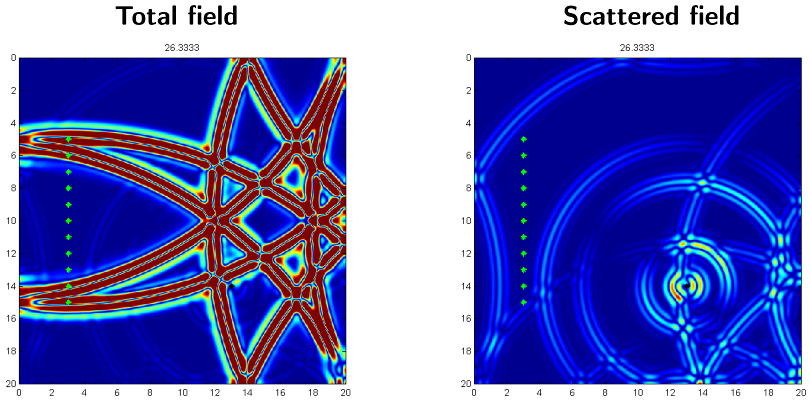


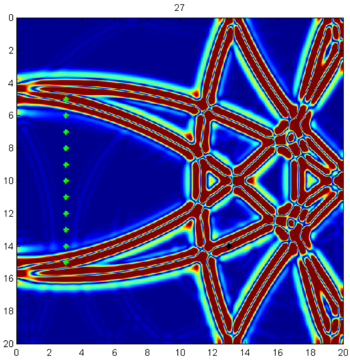

main

# Defect localization - forward step

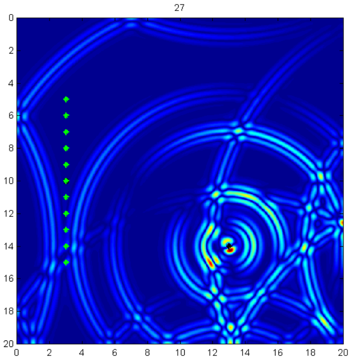


# Defect localization - forward step

**Total field**



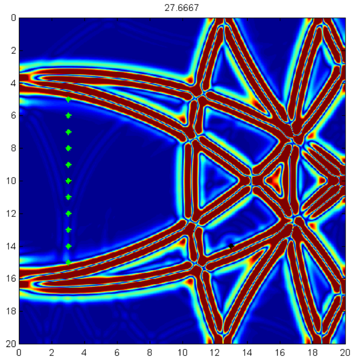

**Scattered field**



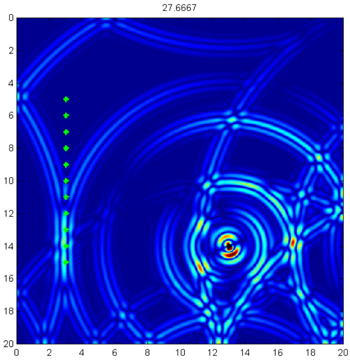

main

# Defect localization - forward step

**Total field**

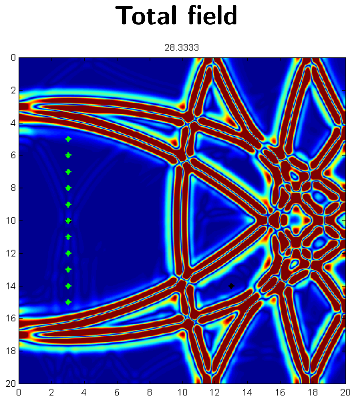


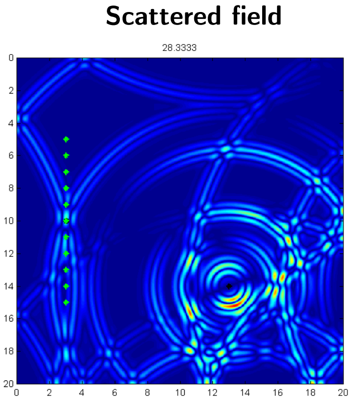

**Scattered field**




main

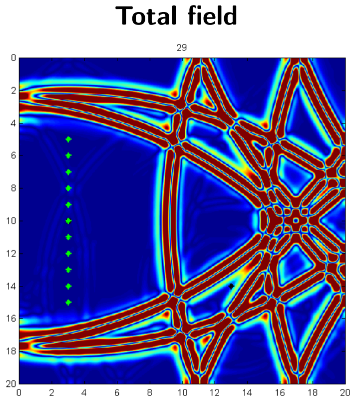
# Defect localization - forward step

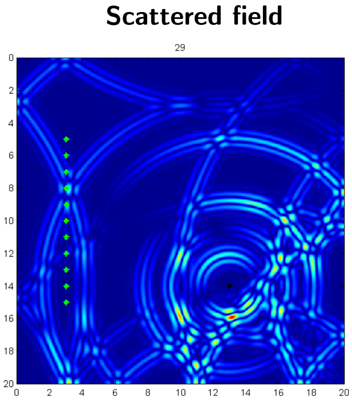



main

# Defect localization - forward step

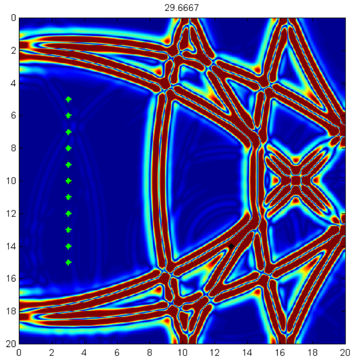


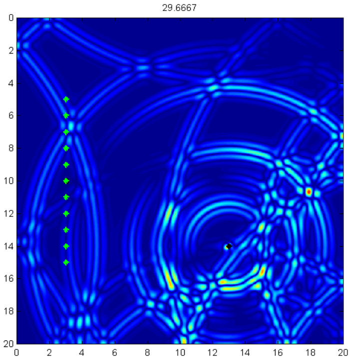

main

# Defect localization - forward step

**Total field**



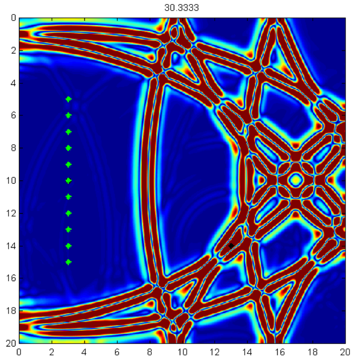

**Scattered field**



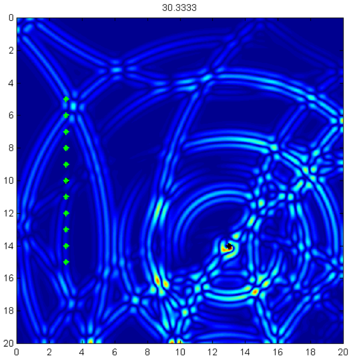

main

# Defect localization - forward step

**Total field**



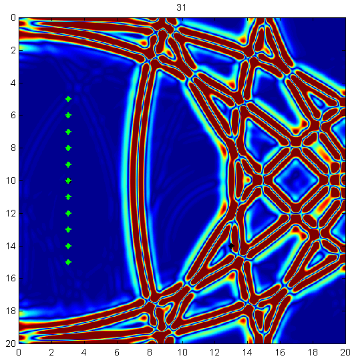

**Scattered field**



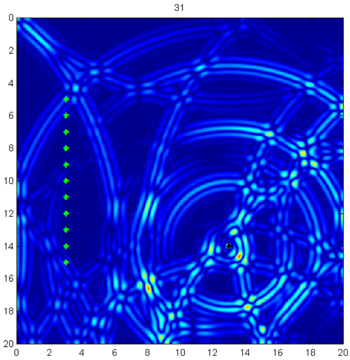

main

# Defect localization - forward step

**Total field**



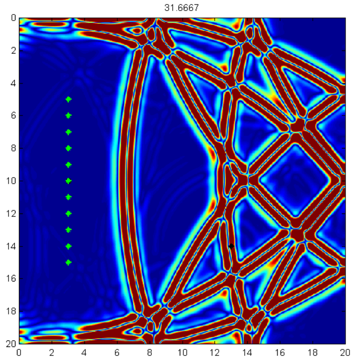

**Scattered field**



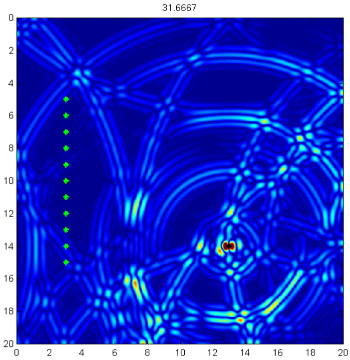

main

# Defect localization - forward step

**Total field**




**Scattered field**

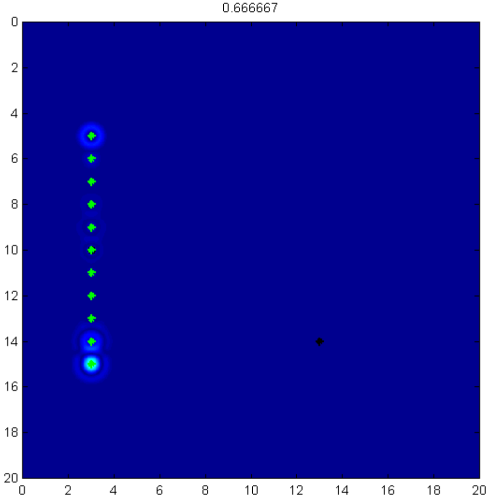



main

- End of forward step

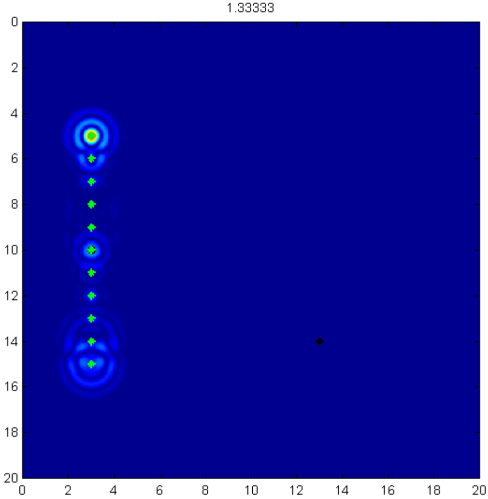
- End of forward step
- Time reverse the recordings and rebroadcast

# Defect localization - backward step

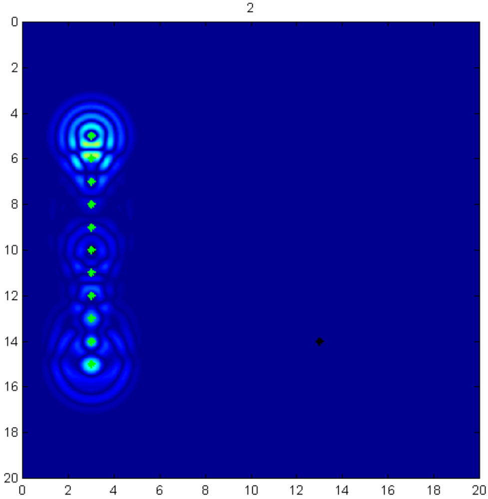



main

# Defect localization - backward step

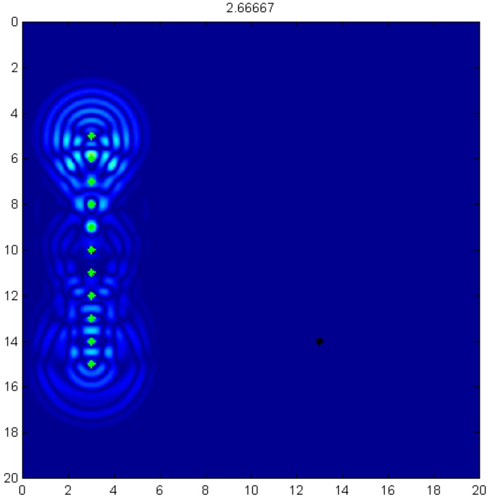


# Defect localization - backward step

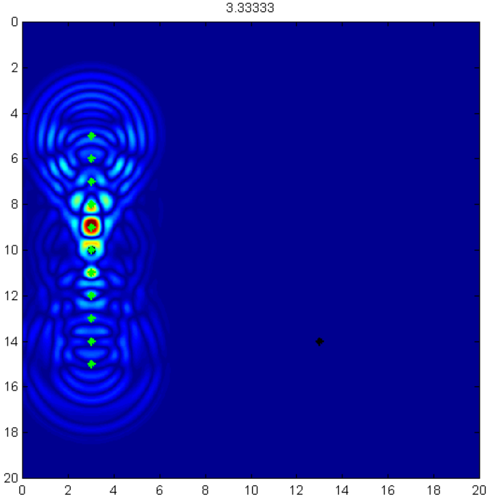



main

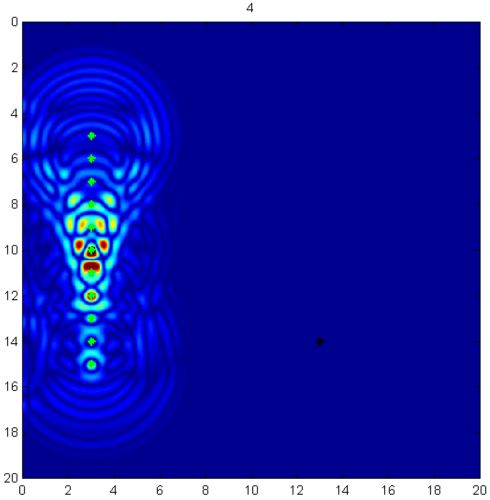
# Defect localization - backward step



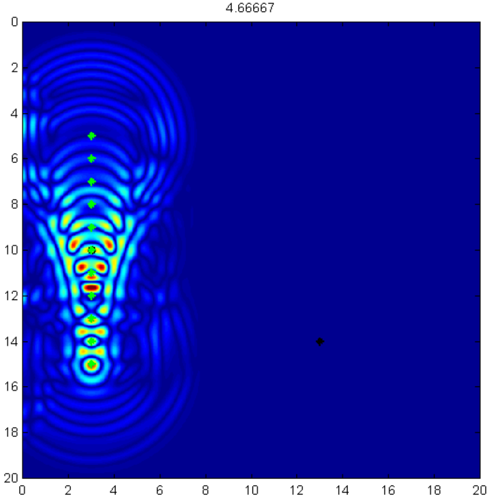
# Defect localization - backward step



# Defect localization - backward step

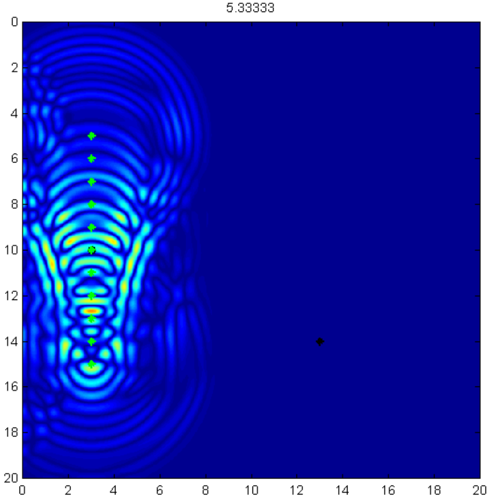


# Defect localization - backward step



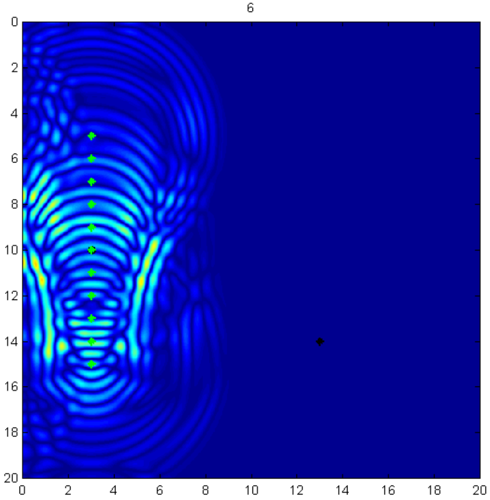

main

# Defect localization - backward step



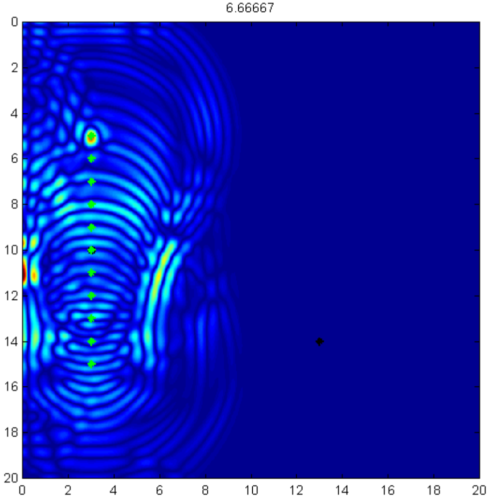

main

# Defect localization - backward step



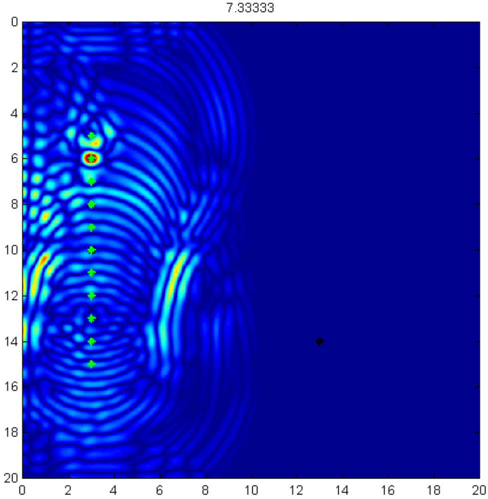

main

# Defect localization - backward step



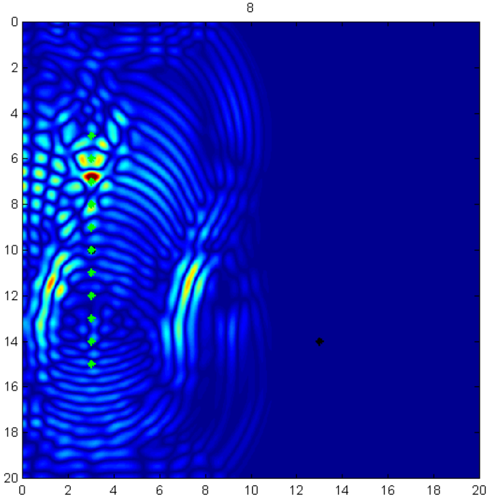

main

# Defect localization - backward step



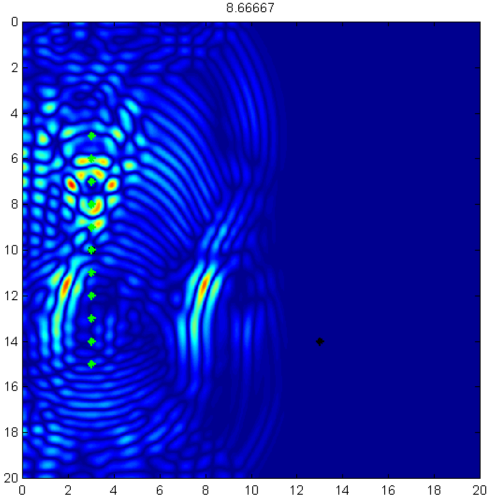

main

# Defect localization - backward step



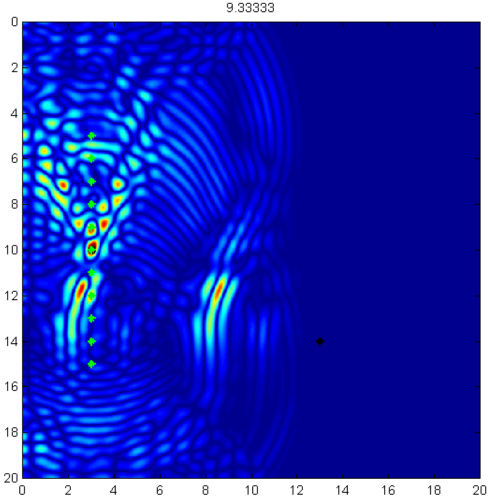

main

# Defect localization - backward step



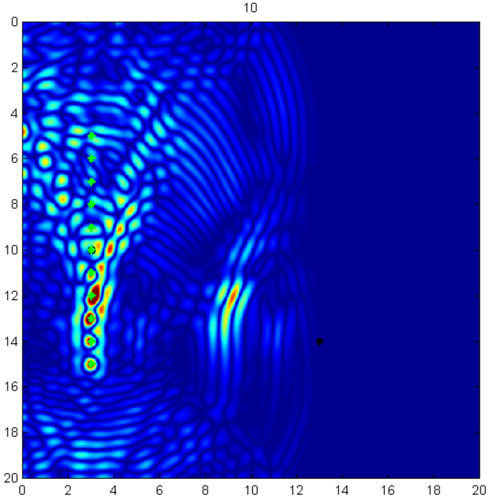

main

# Defect localization - backward step



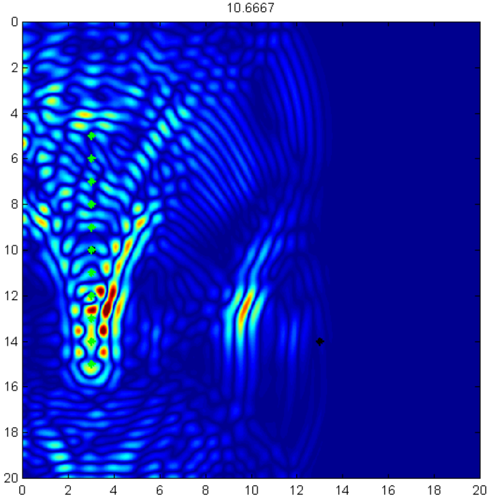

main

# Defect localization - backward step



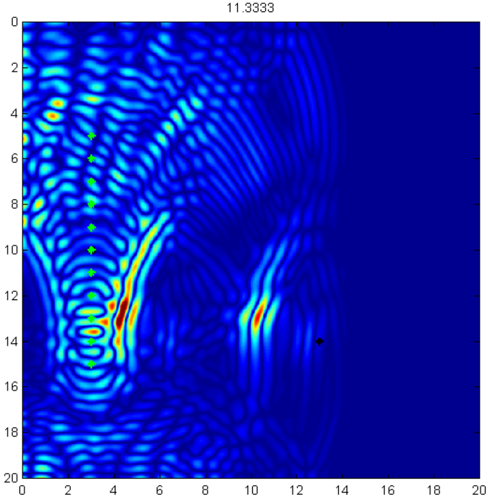

main

# Defect localization - backward step



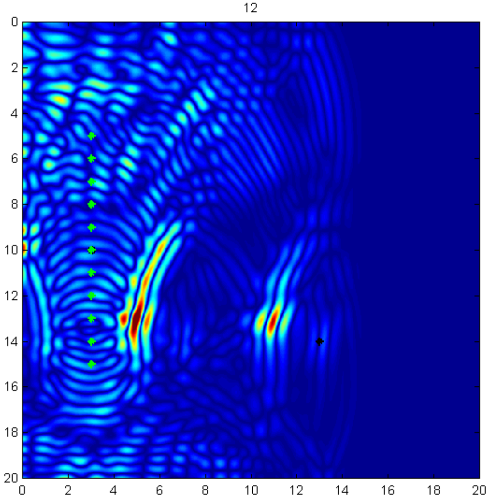

main

# Defect localization - backward step



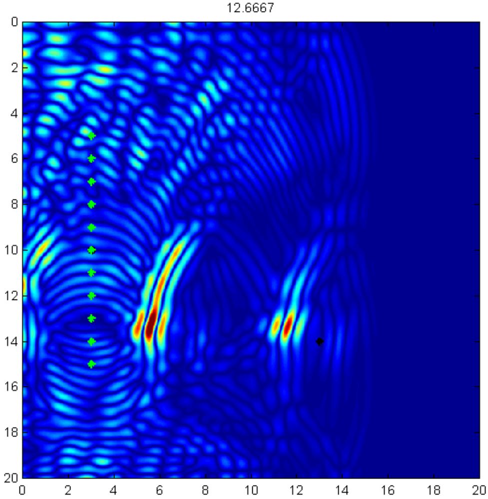

main

# Defect localization - backward step



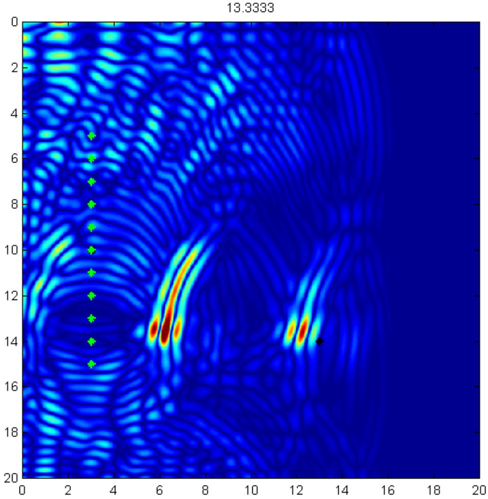

main

# Defect localization - backward step

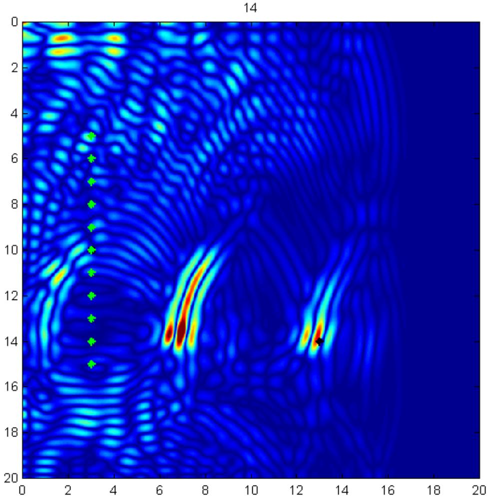



main

# Defect localization - backward step

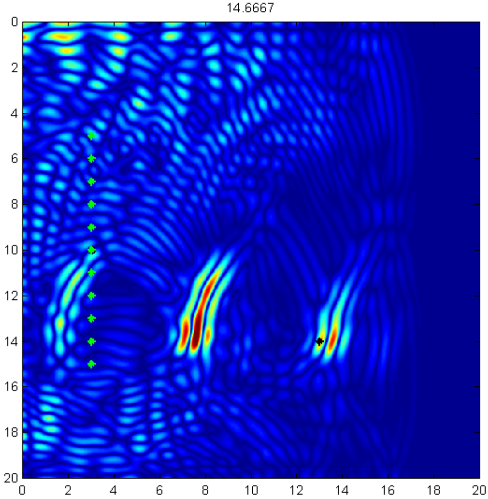


# Defect localization - backward step



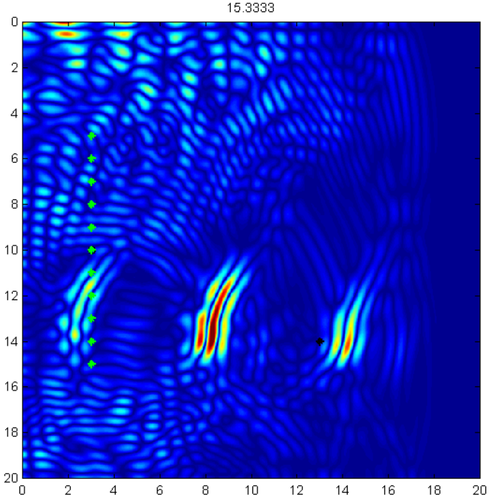

main

# Defect localization - backward step

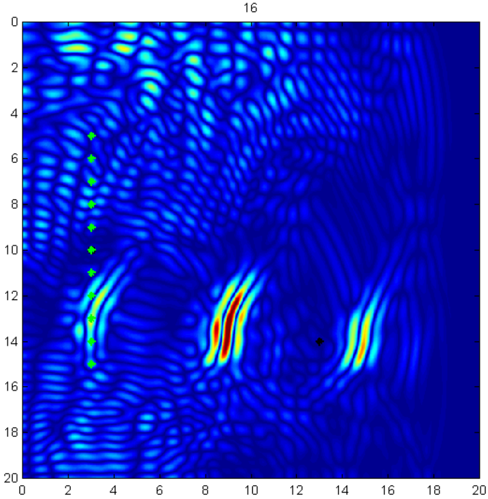



main

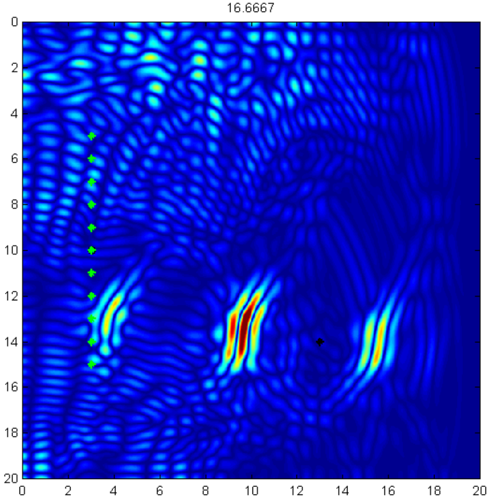
# Defect localization - backward step



# Defect localization - backward step

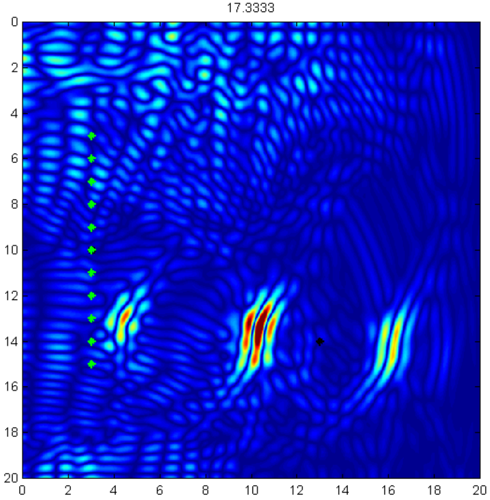


# Defect localization - backward step



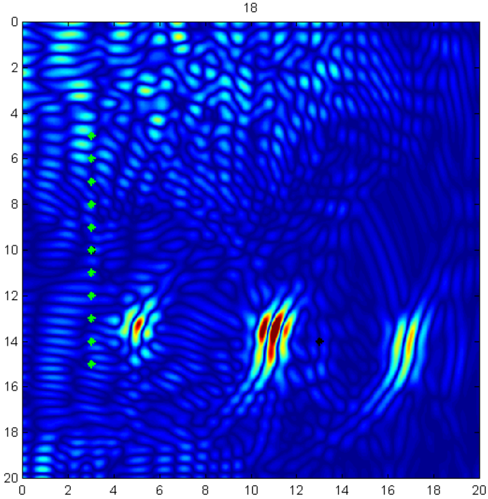

main

# Defect localization - backward step



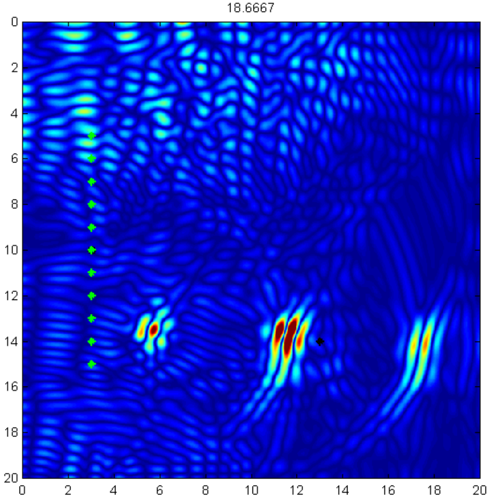

main

# Defect localization - backward step

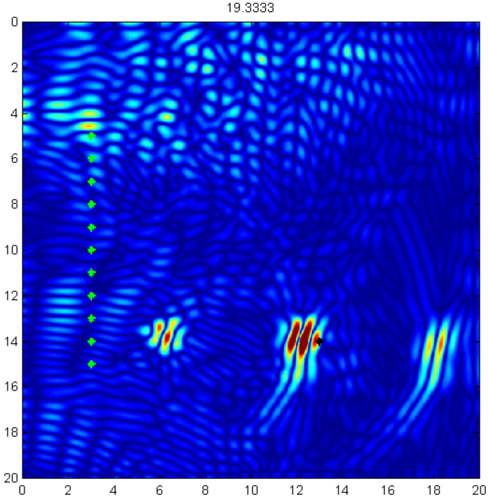



main

# Defect localization - backward step

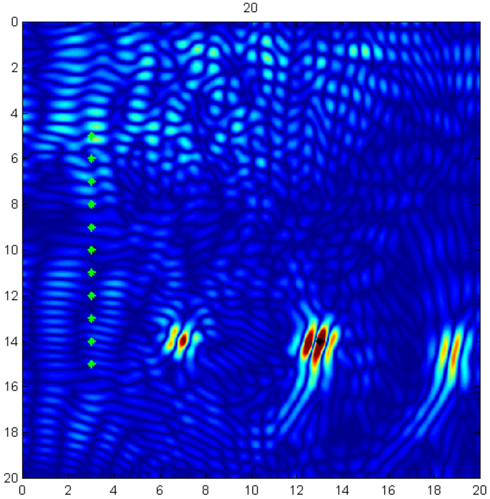


# Defect localization - backward step




main

# Defect localization - backward step

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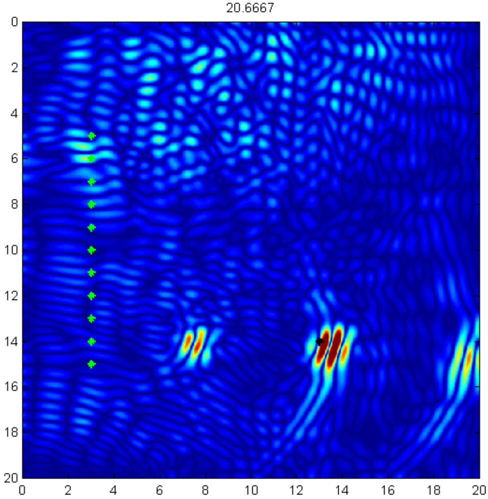
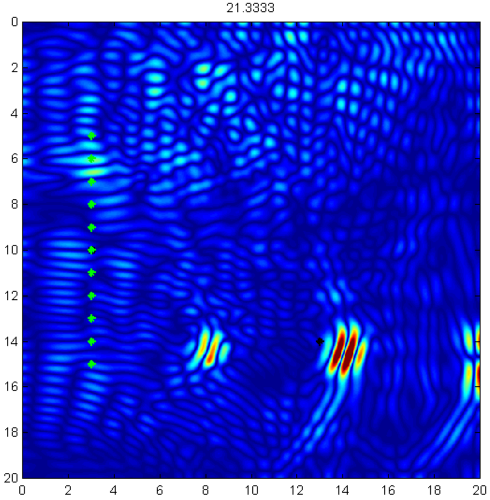
# Defect localization - backward step



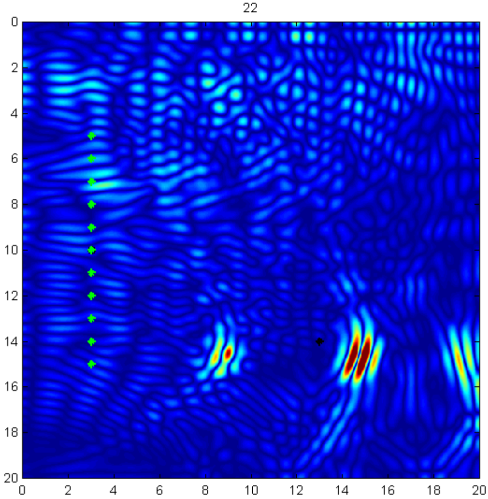

main

# Defect localization - backward step




main

# Defect localization - backward step

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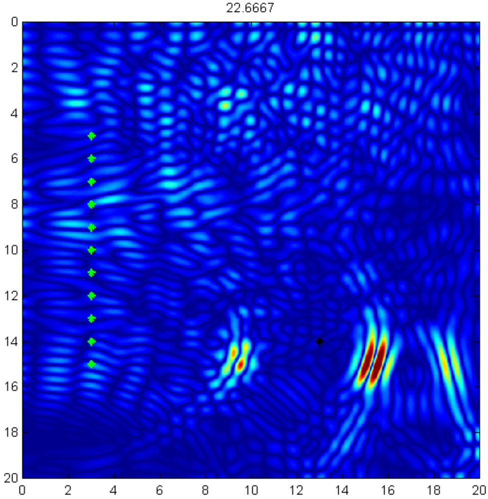
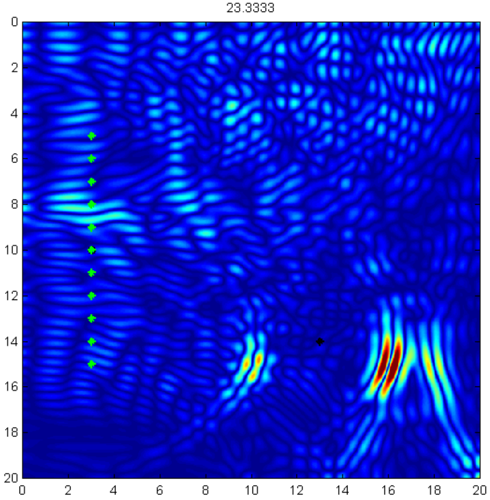
# Defect localization - backward step



# Defect localization - backward step

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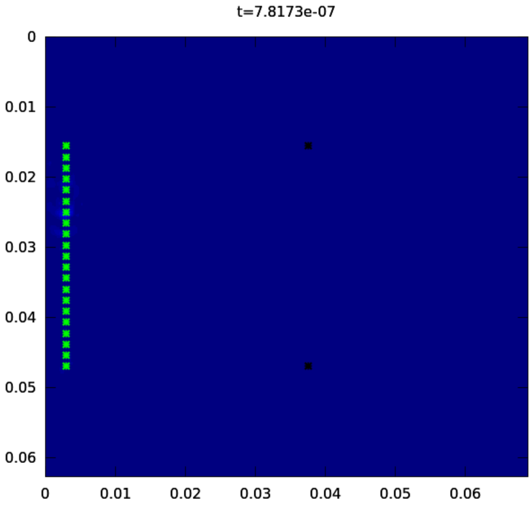
# Backward step

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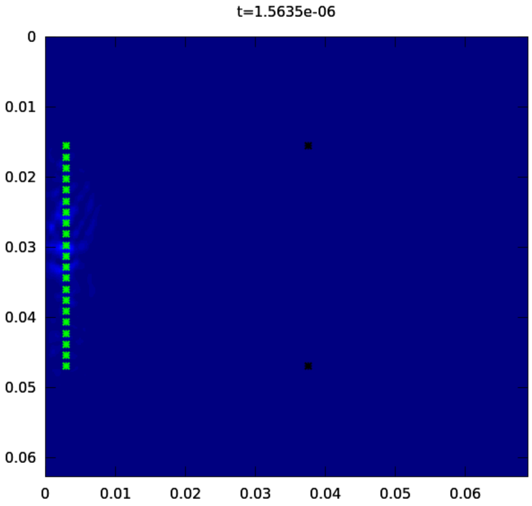
# Backward step

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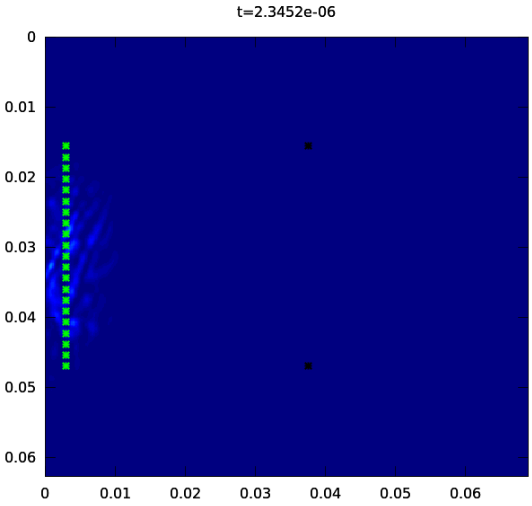
# Backward step



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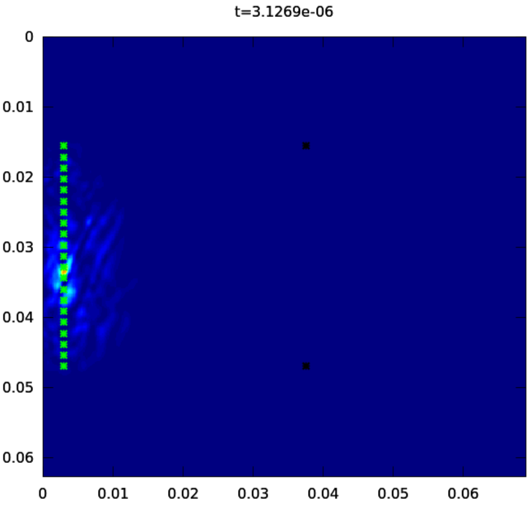


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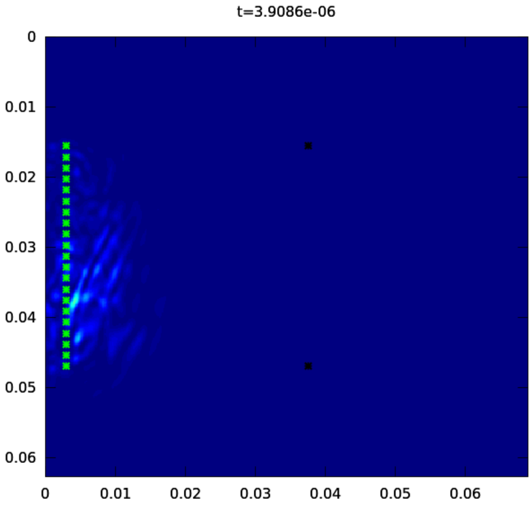
# Backward step

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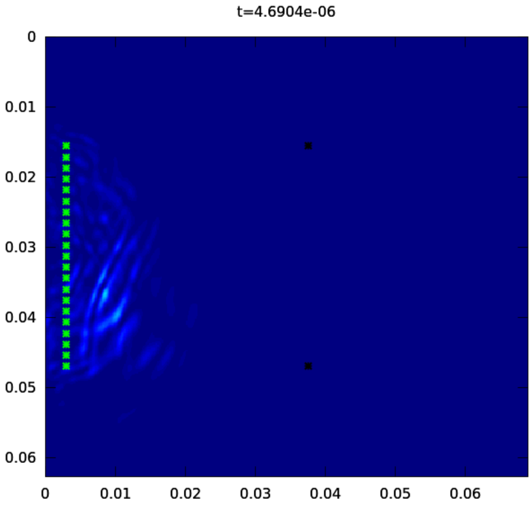
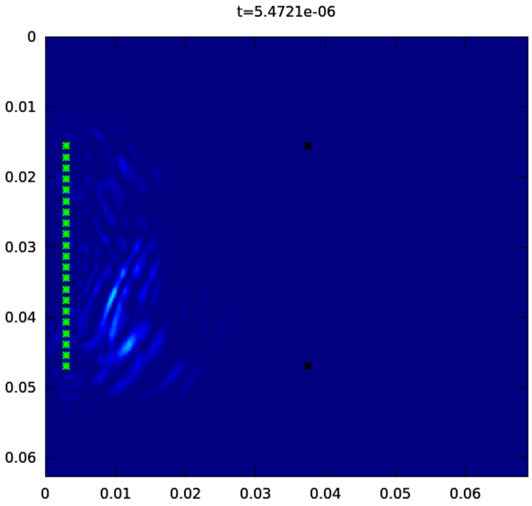
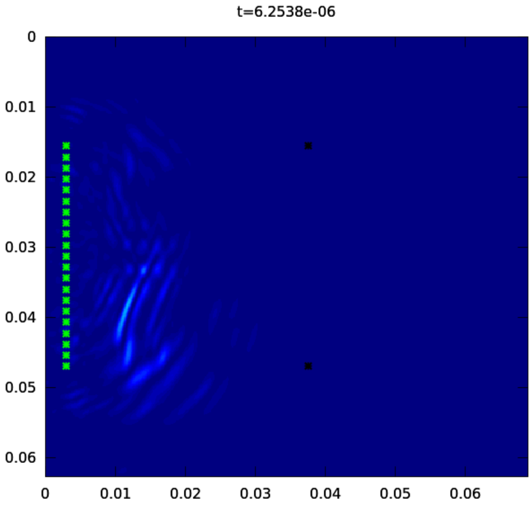
# Backward step



# Backward step



# Backward step

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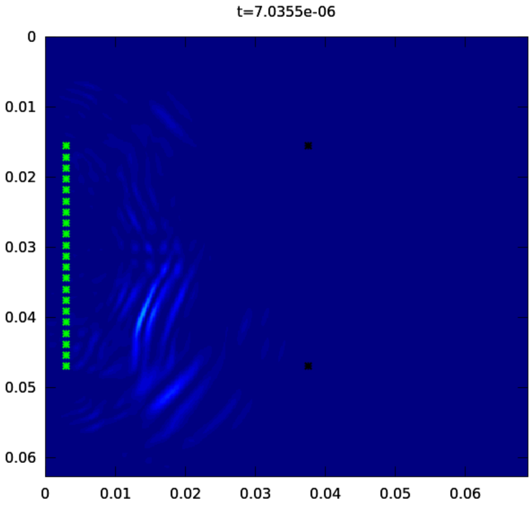
# Backward step

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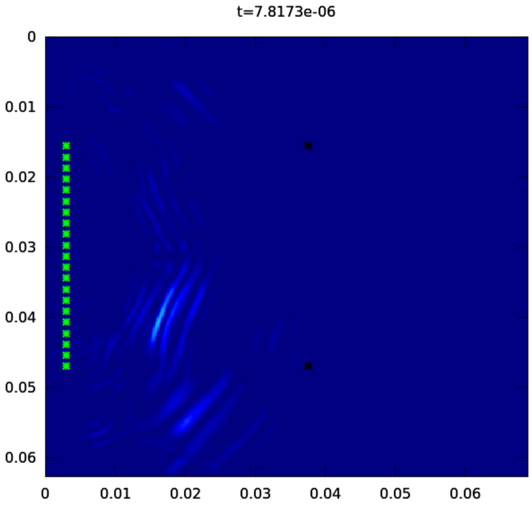
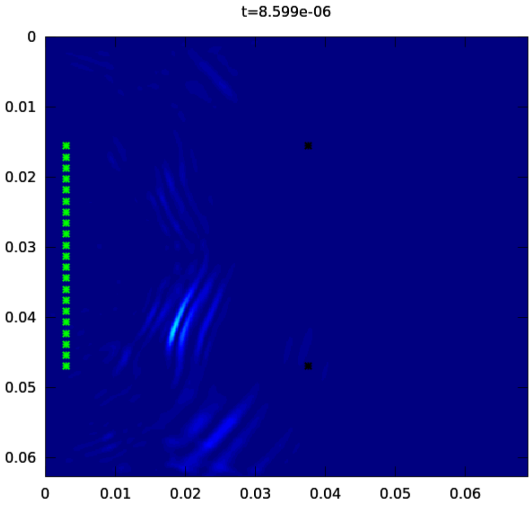
# Backward step




main

# Backward step

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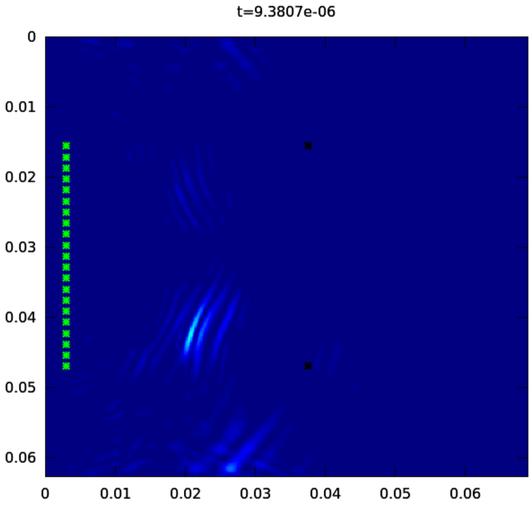
# Backward step

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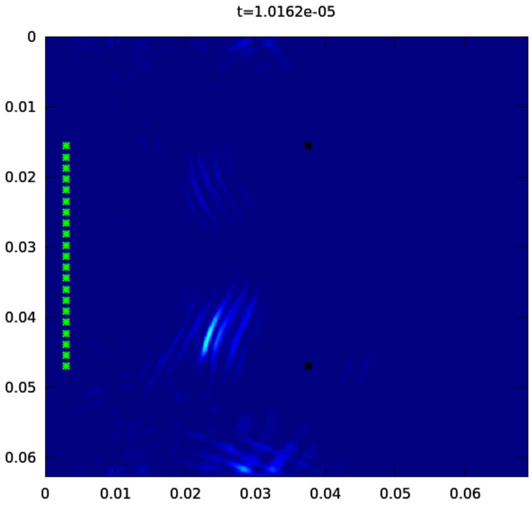
# Backward step

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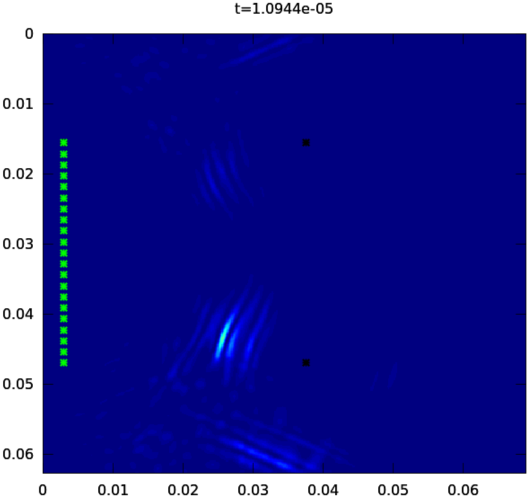
# Backward step

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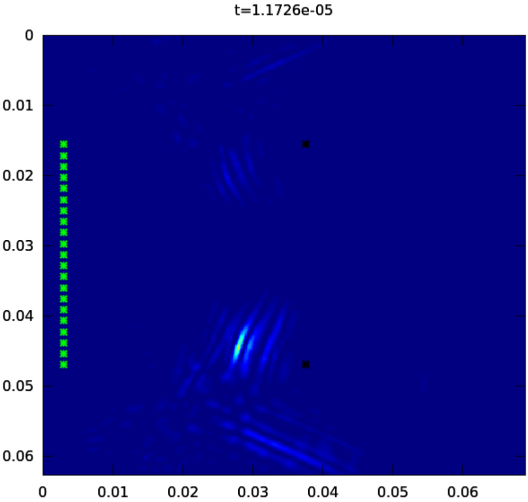
# Backward step

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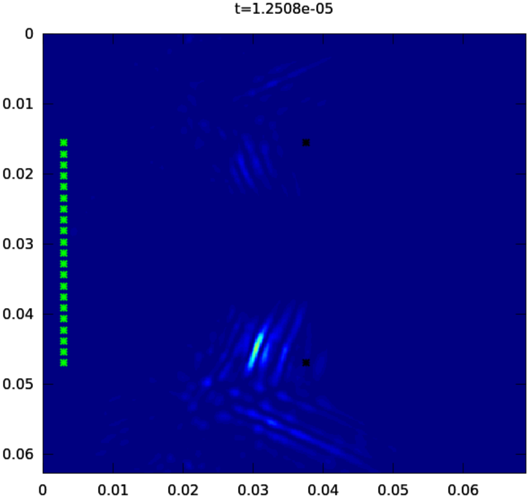
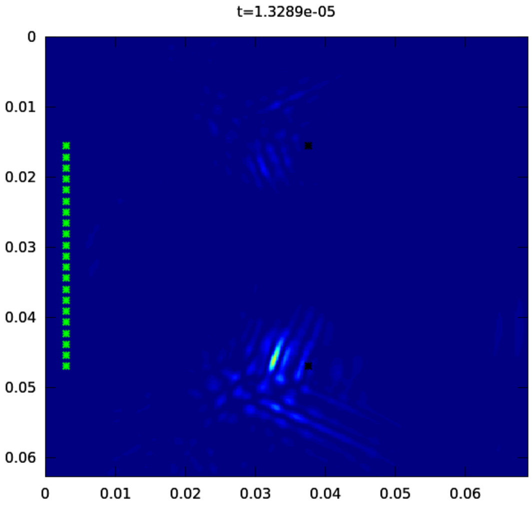
# Backward step



# Backward step

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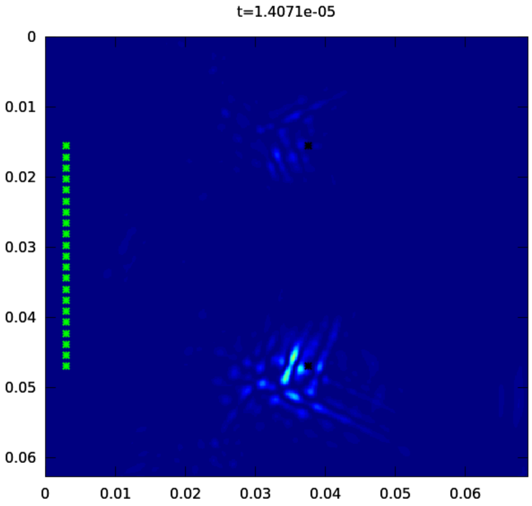
# Backward step

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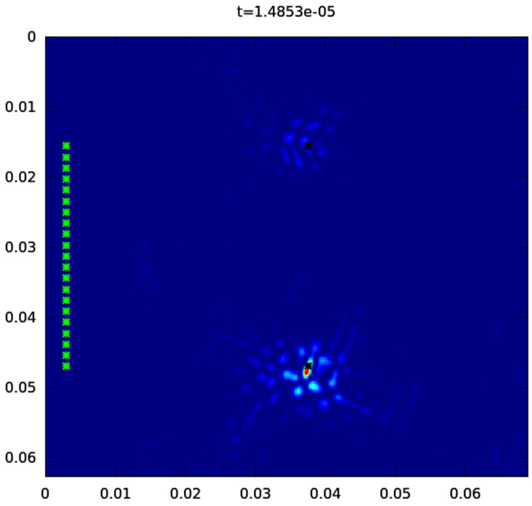
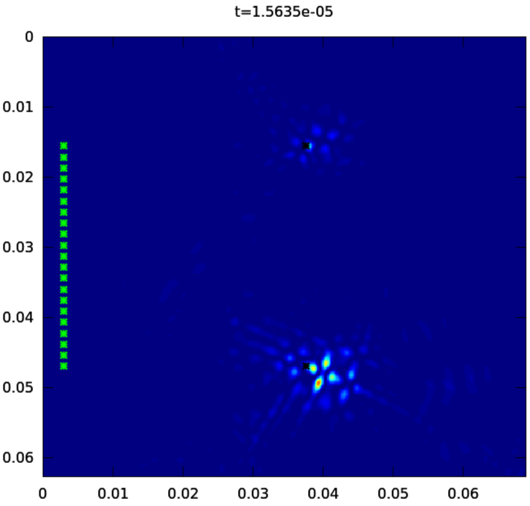
# Backward step



# Backward step

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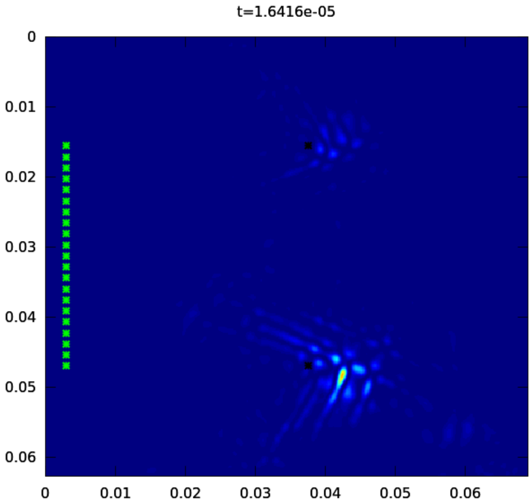
# Backward step

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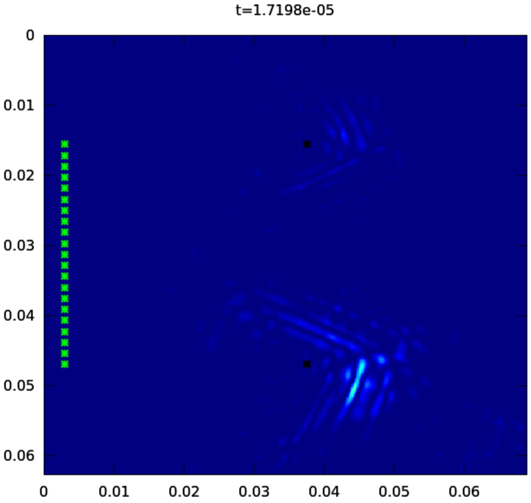
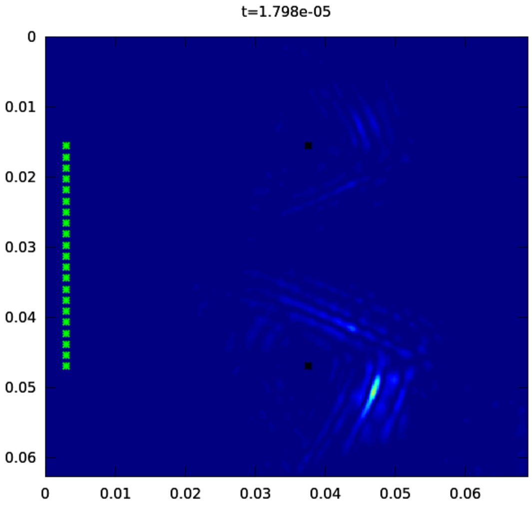
# Backward step

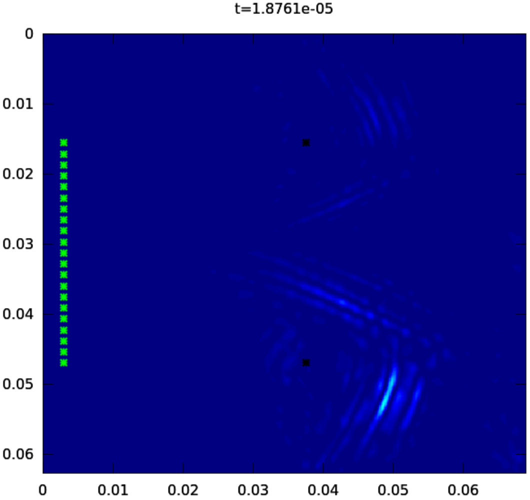



main

# Backward step

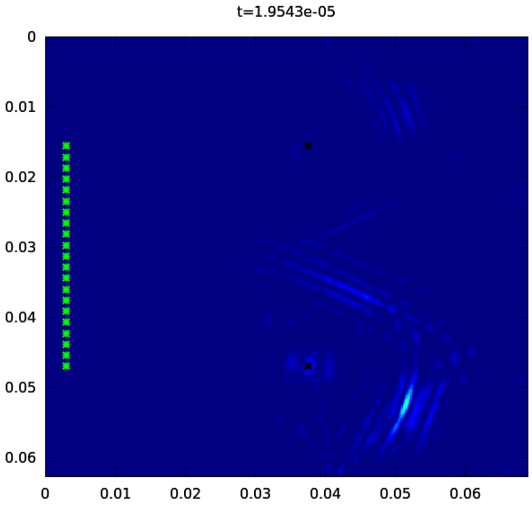


# Backward step



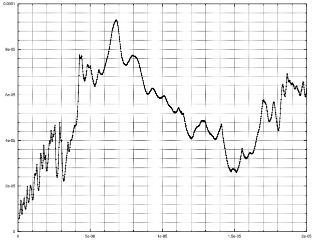

main

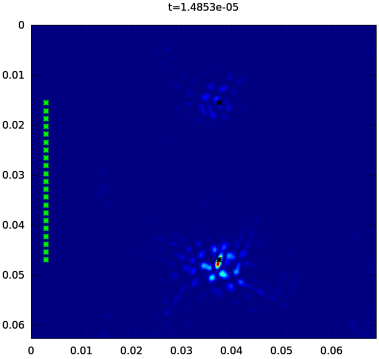
# Backward step



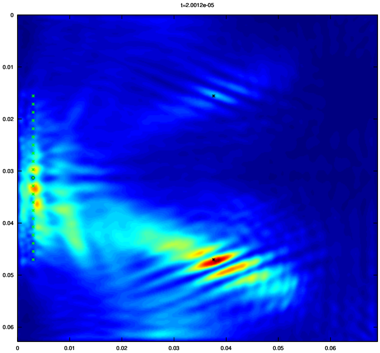

main

$$\mathcal{E}(t) = \frac{1}{2}(A\sigma, \sigma) + \frac{1}{2}(\rho v, v)$$

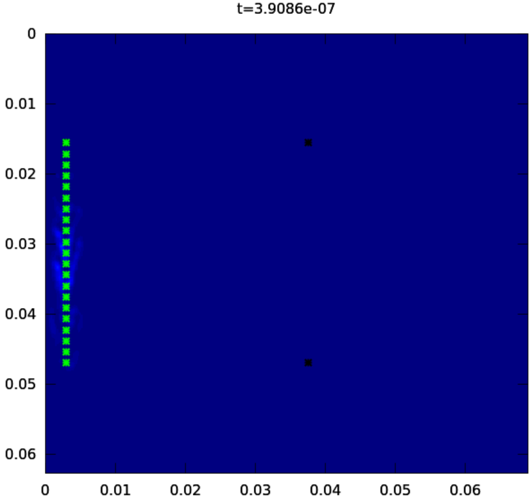



$$\mathcal{I}(\mathbf{y}) = \int_0^T \mathcal{E}(\mathbf{y}, t)\mathrm{d}t, \qquad \mathbf{y} \in \Omega$$

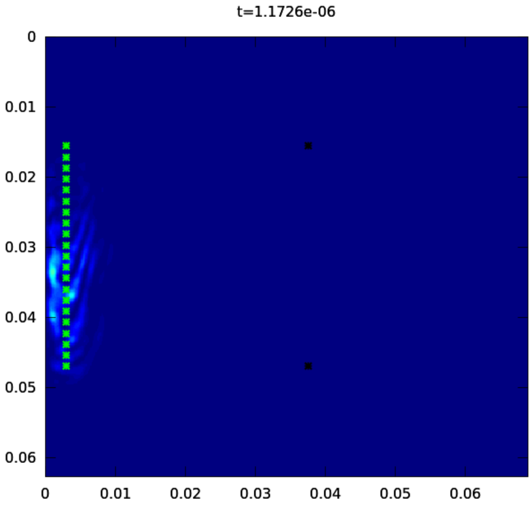


# Backward step - first singular value



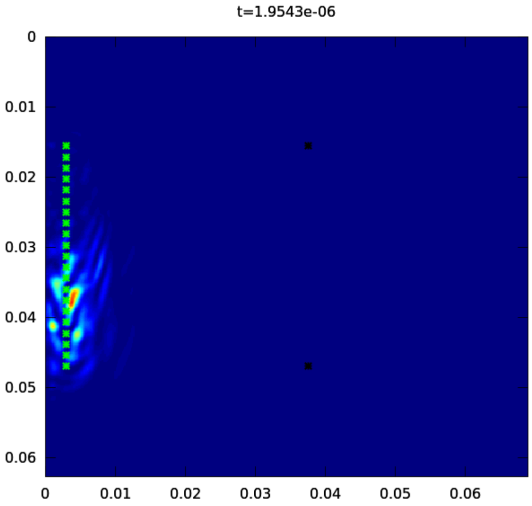

main

# Backward step - first singular value

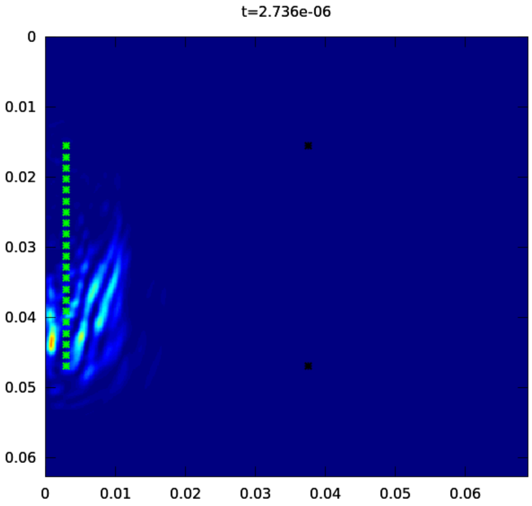



main

# Backward step - first singular value

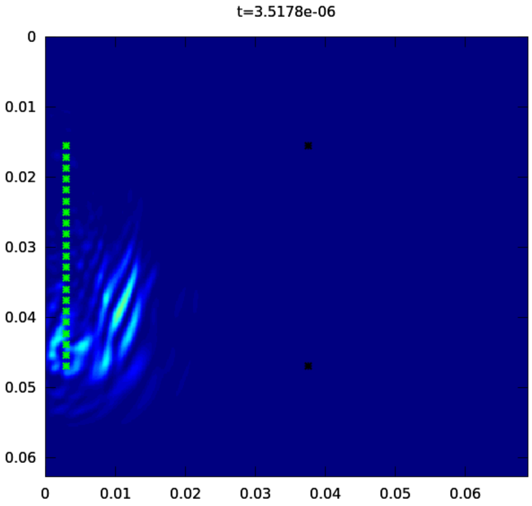


# Backward step - first singular value



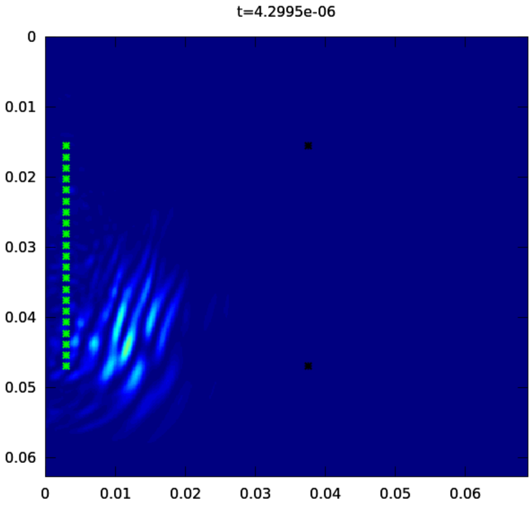

main

# Backward step - first singular value



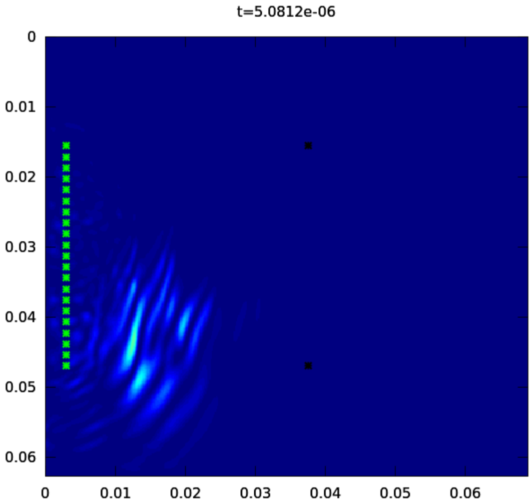

main

# Backward step - first singular value



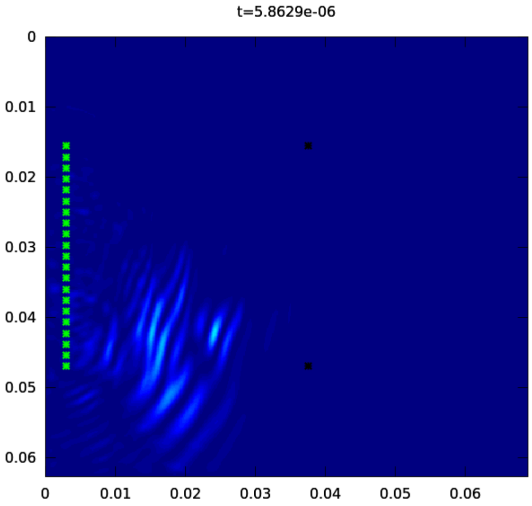

main

# Backward step - first singular value




main

# Backward step - first singular value

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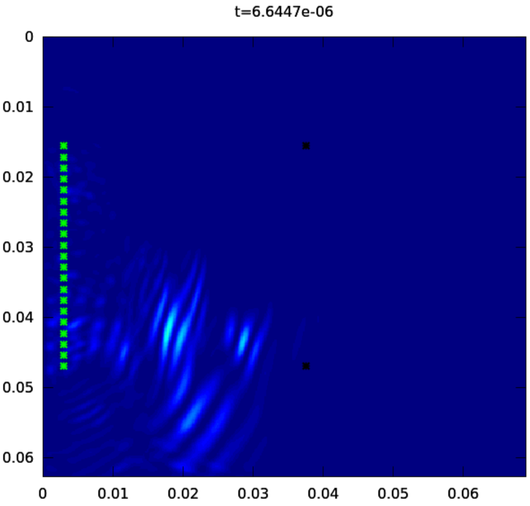
# Backward step - first singular value

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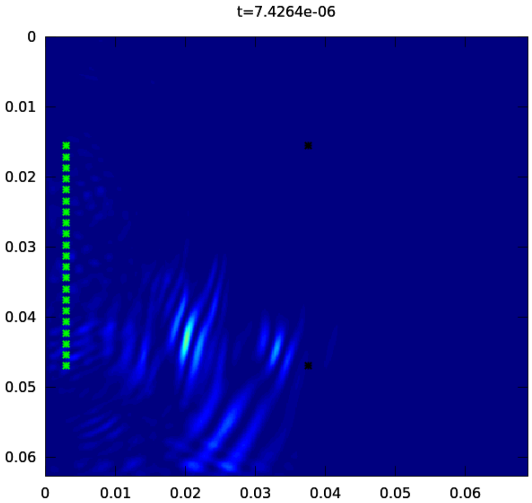
# Backward step - first singular value

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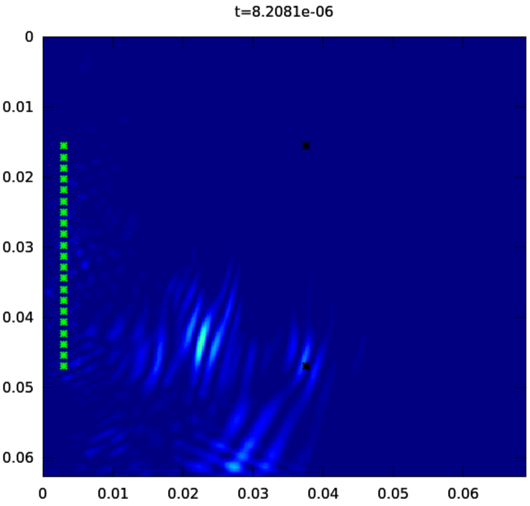
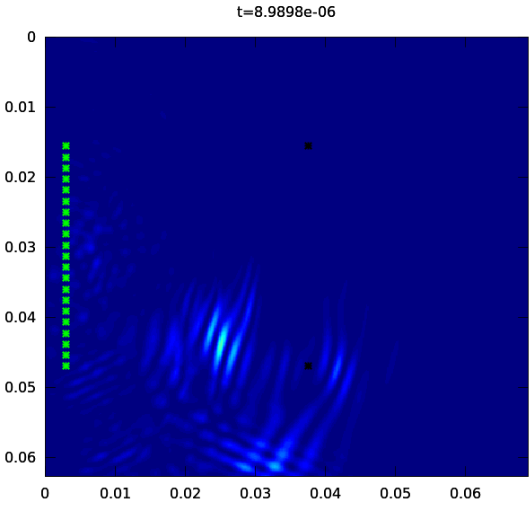
# Backward step - first singular value



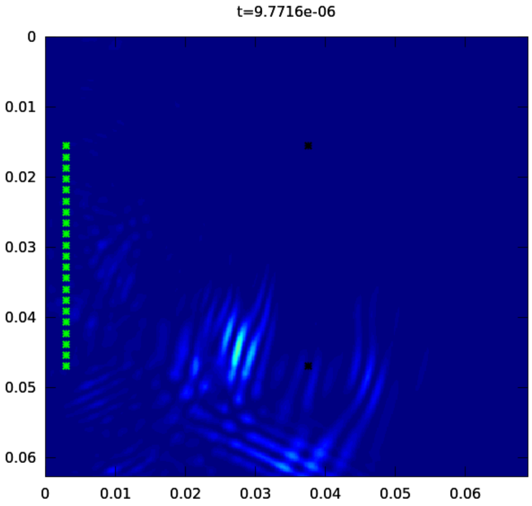

main

# Backward step - first singular value




main

# Backward step - first singular value

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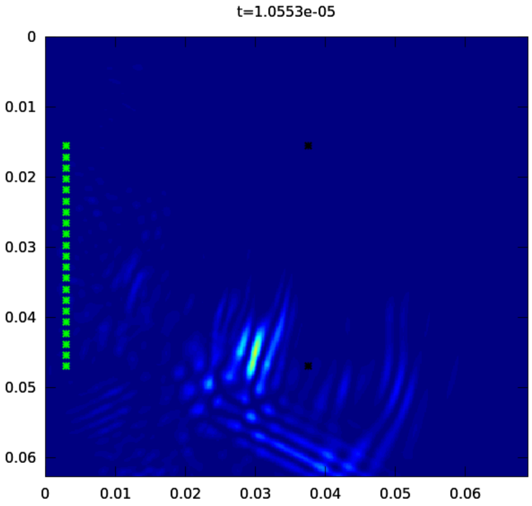
# Backward step - first singular value

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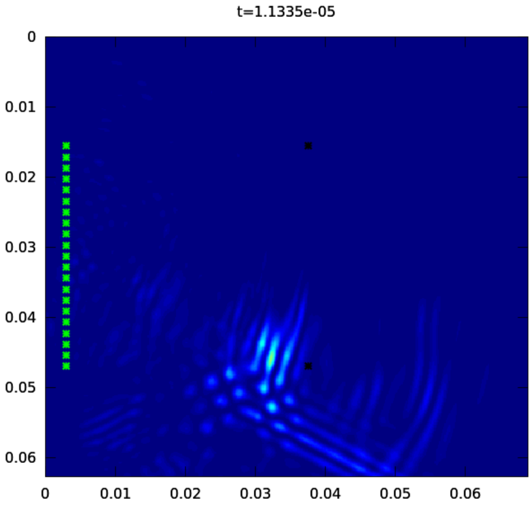
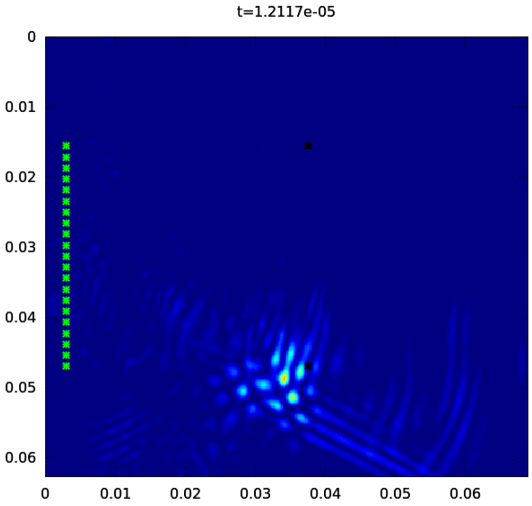
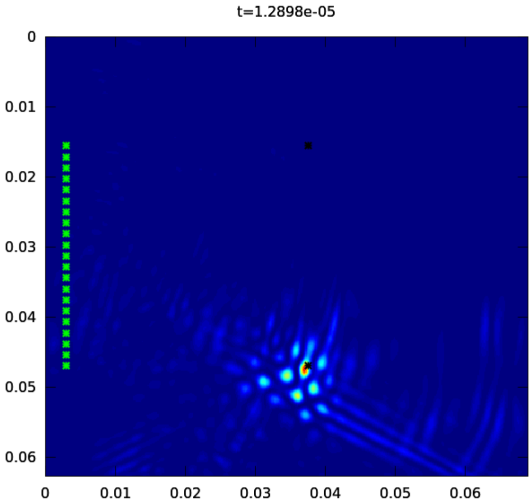
# Backward step - first singular value



# Backward step - first singular value



# Backward step - first singular value

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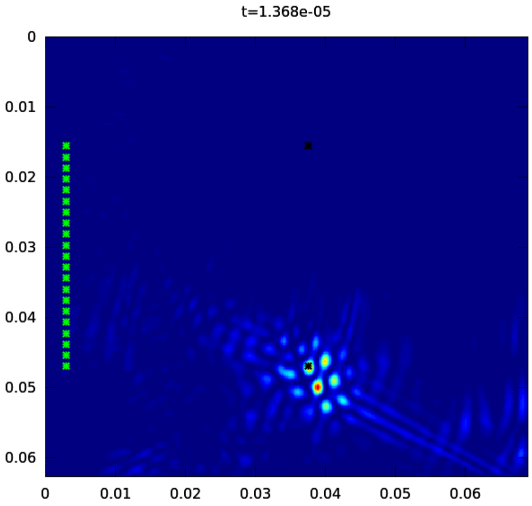
# Backward step - first singular value

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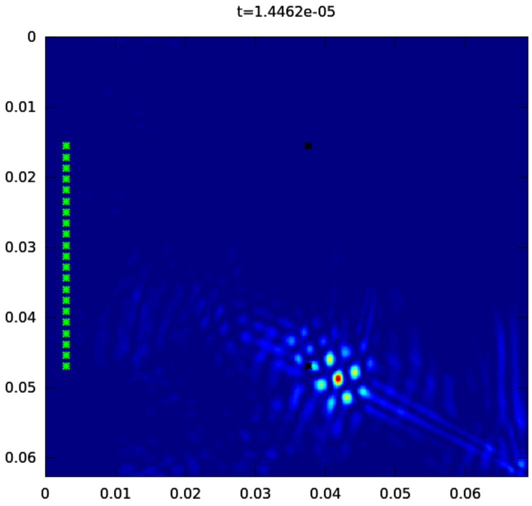
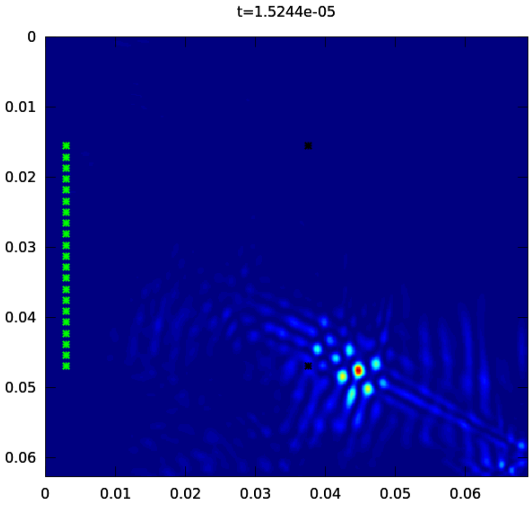
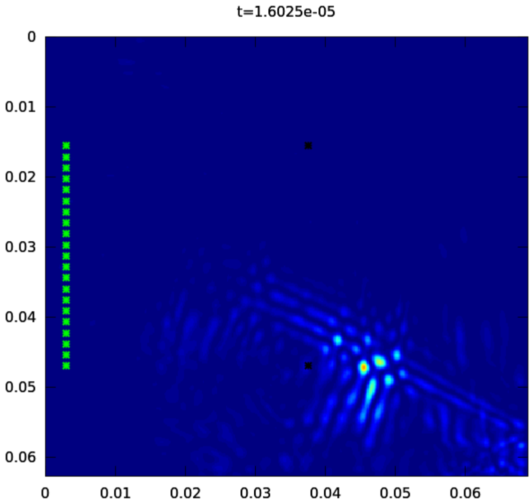
# Backward step - first singular value



# Backward step - first singular value

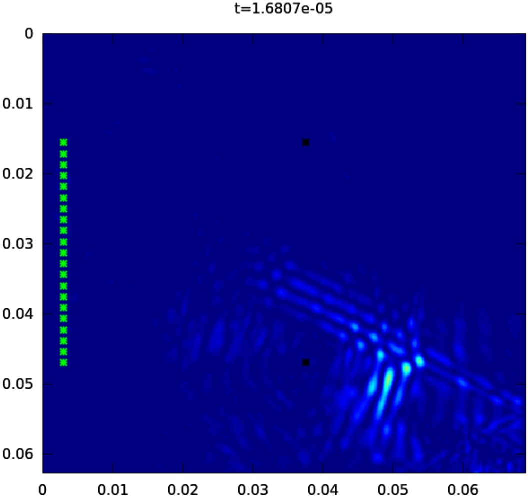


# Backward step - first singular value



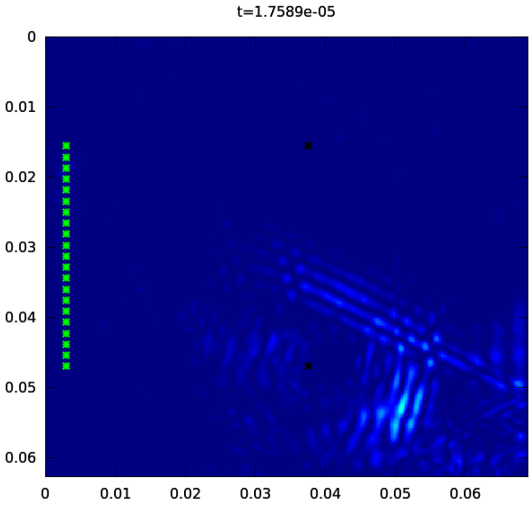

main

# Backward step - first singular value

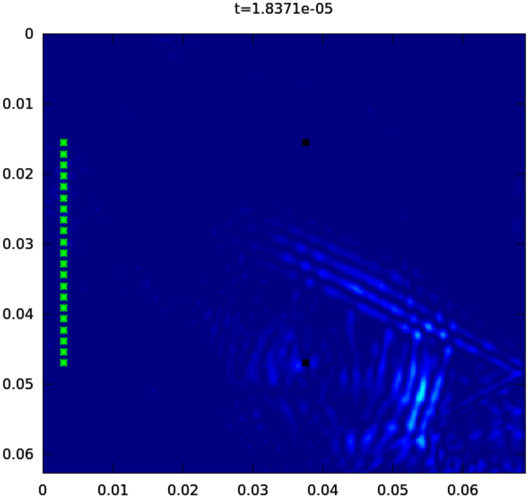



main

# Backward step - first singular value

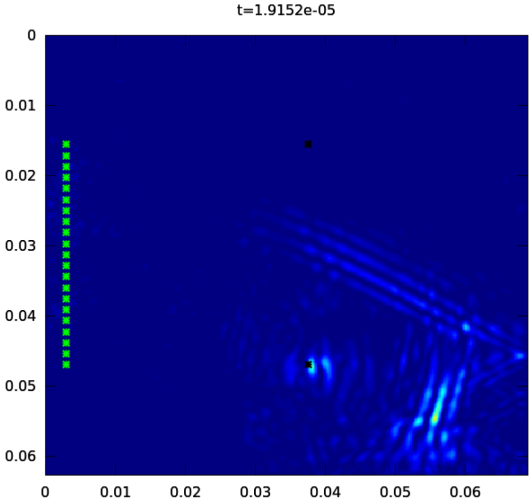


# Backward step - first singular value




main

# Backward step - first singular value

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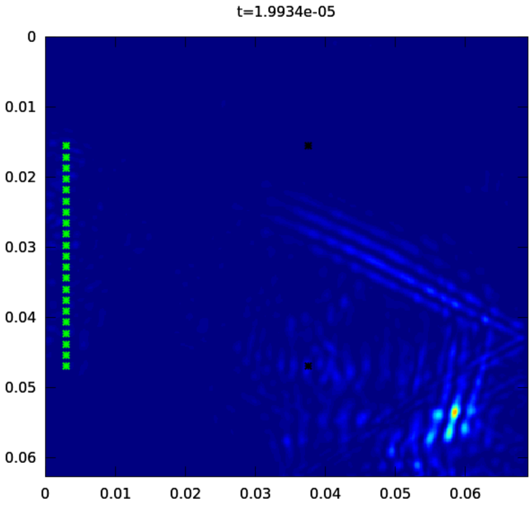
# Backward step - first singular value

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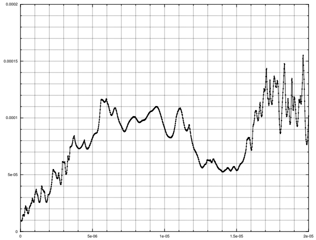
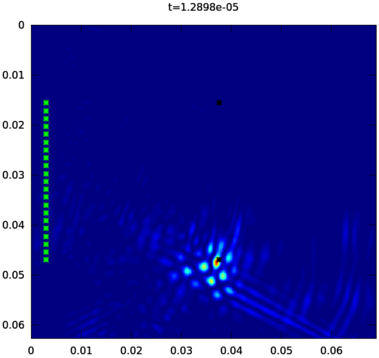
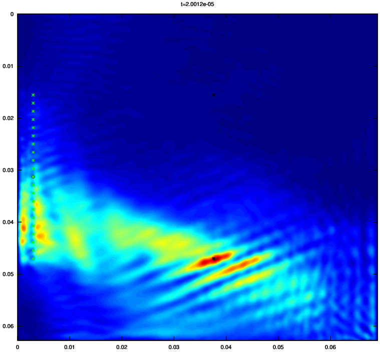
$$\mathcal{E}(t) = \frac{1}{2}(A\sigma, \sigma) + \frac{1}{2}(\rho v, v)$$



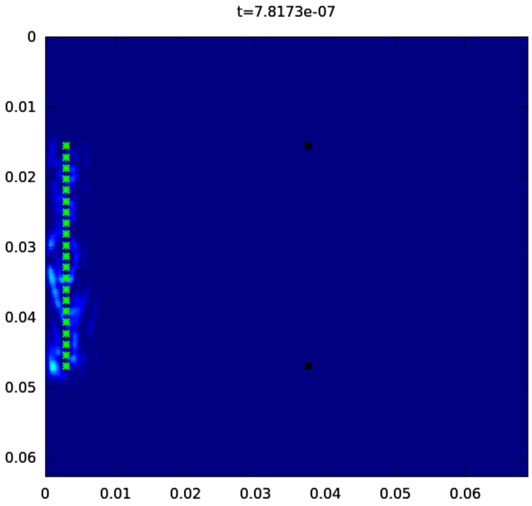
$$\mathcal{I}(\mathbf{y}) = \int_0^T \mathcal{E}(\mathbf{y}, t)\mathrm{d}t, \qquad \mathbf{y} \in \Omega$$



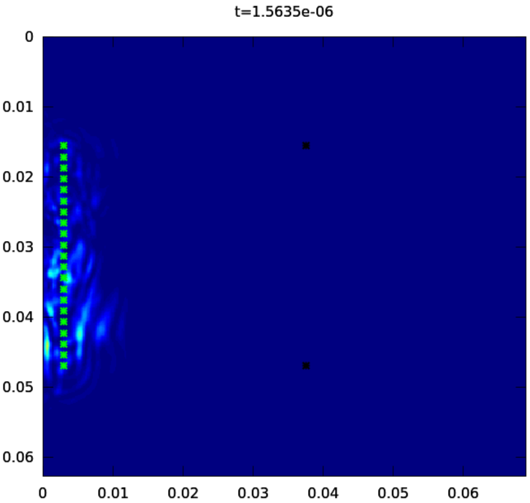




main

# Backward step - second singular value



# Backward step - second singular value

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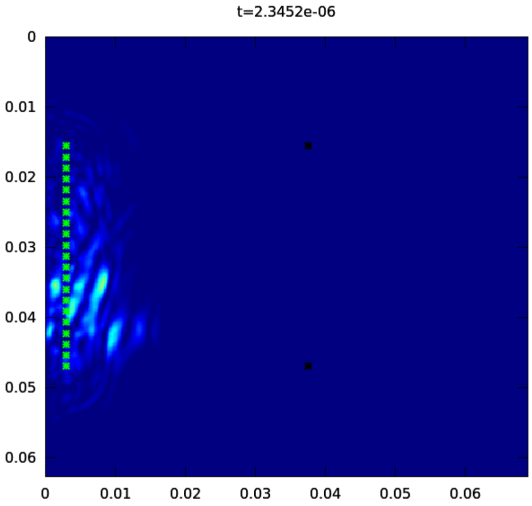
# Backward step - second singular value

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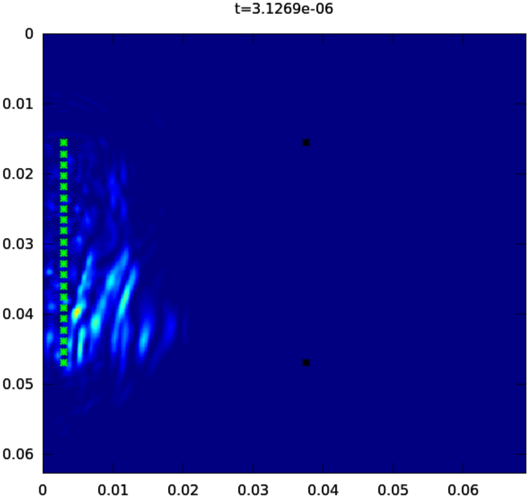
# Backward step - second singular value

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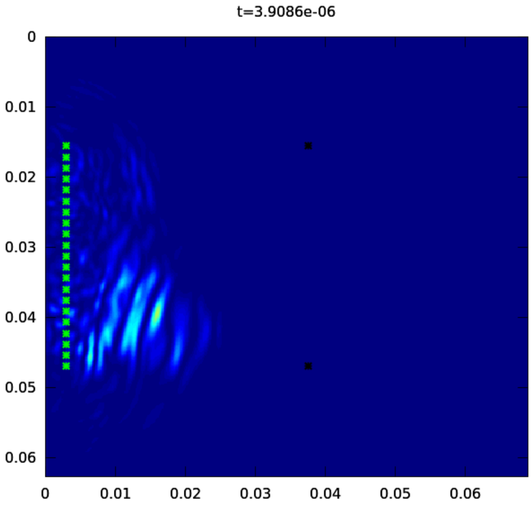
# Backward step - second singular value

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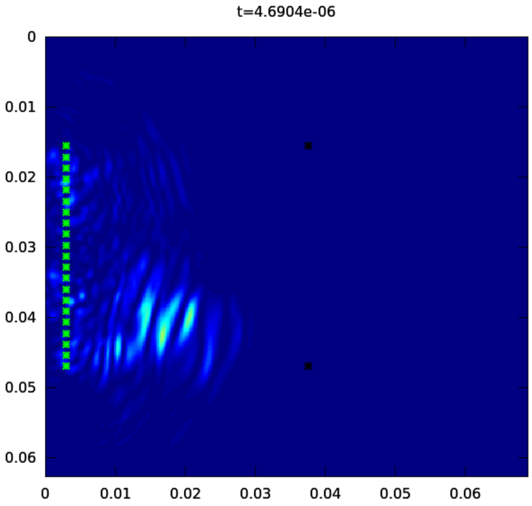


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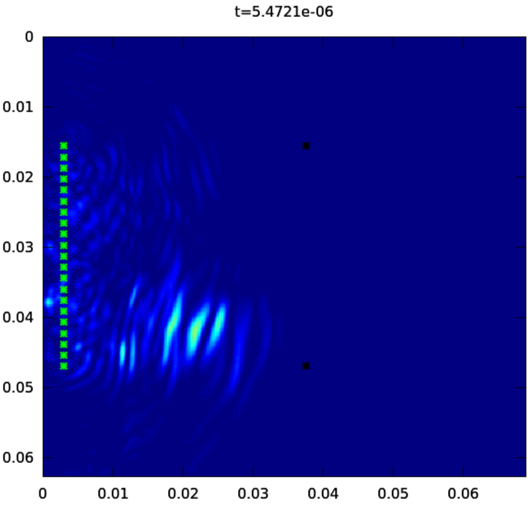
# Backward step - second singular value

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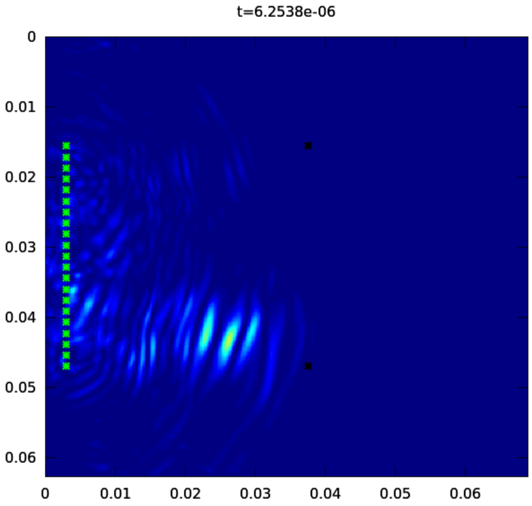
# Backward step - second singular value

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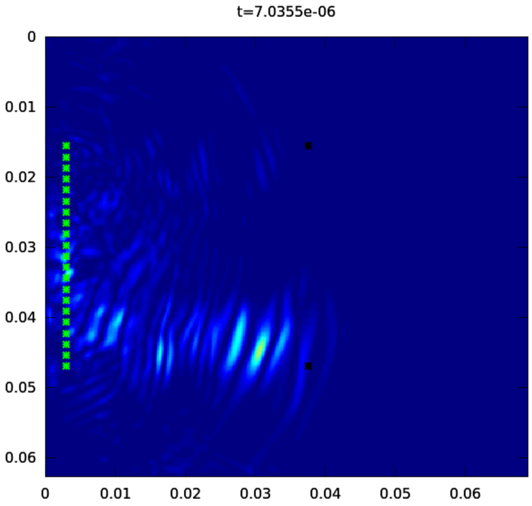
# Backward step - second singular value



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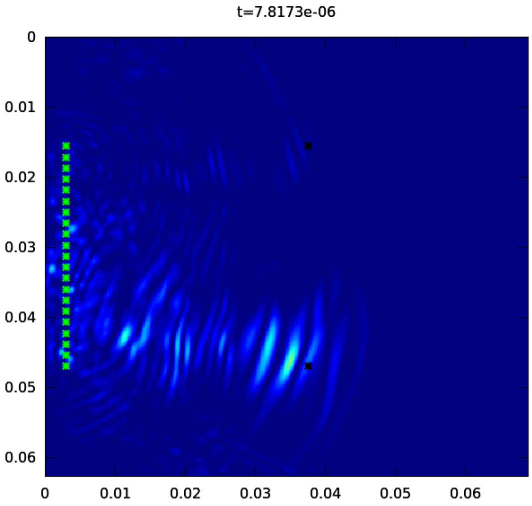


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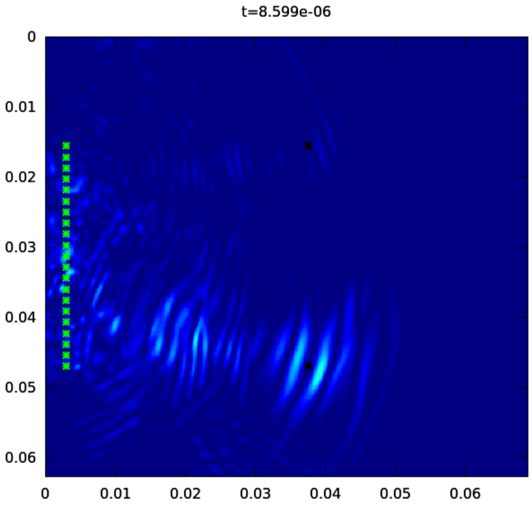
# Backward step - second singular value

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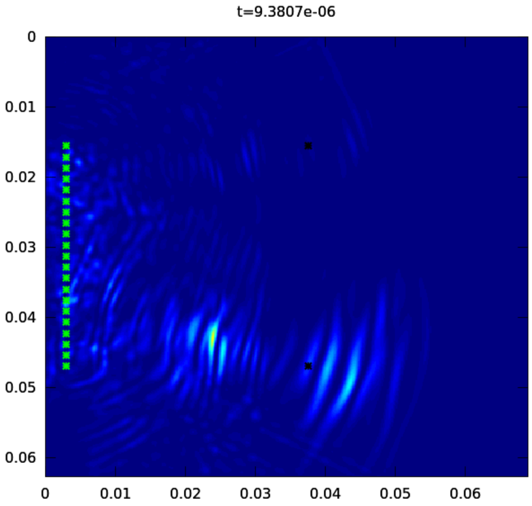
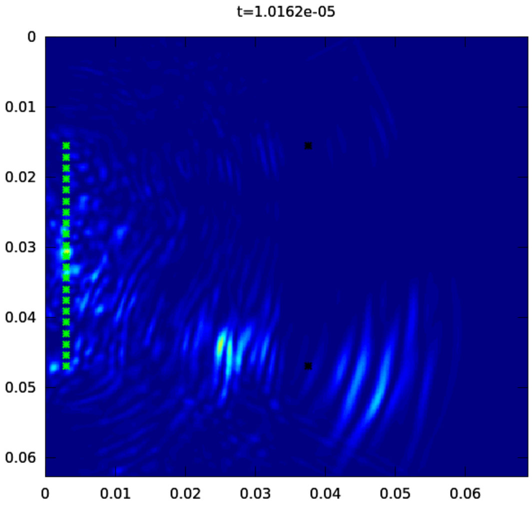
# Backward step - second singular value



# Backward step - second singular value

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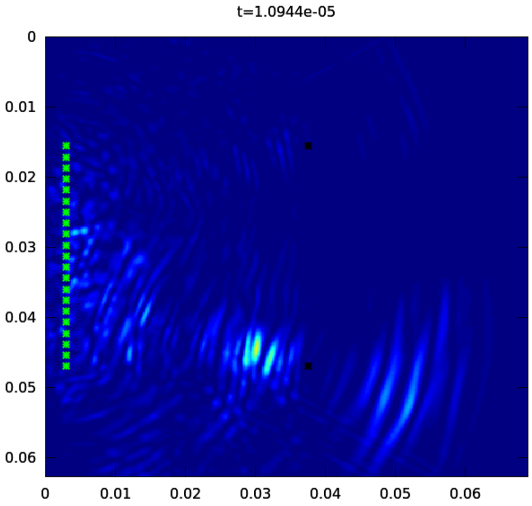
# Backward step - second singular value

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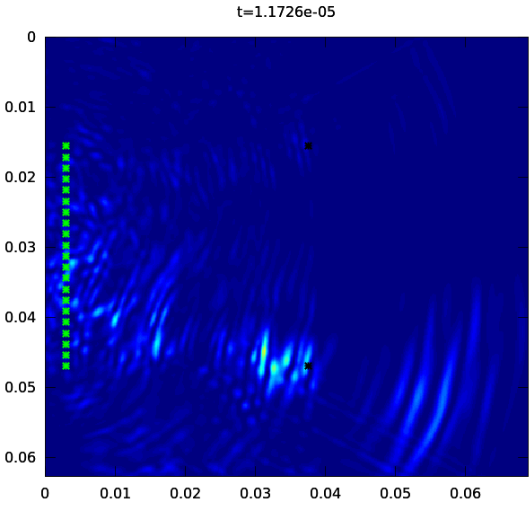
# Backward step - second singular value

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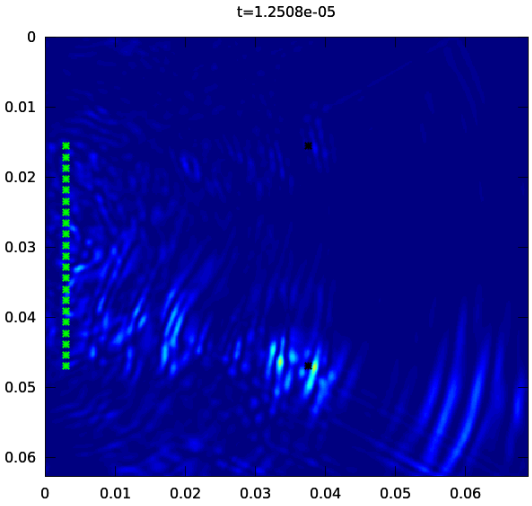
# Backward step - second singular value

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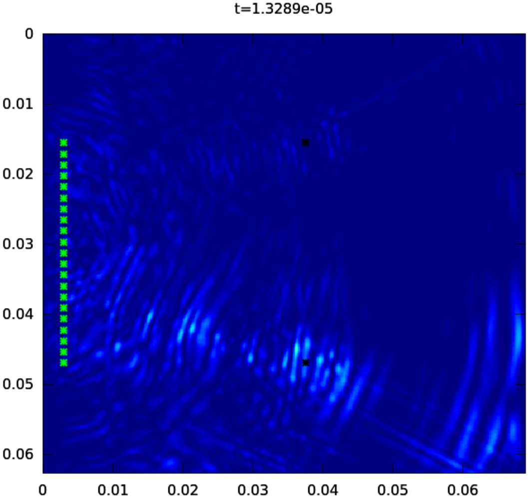


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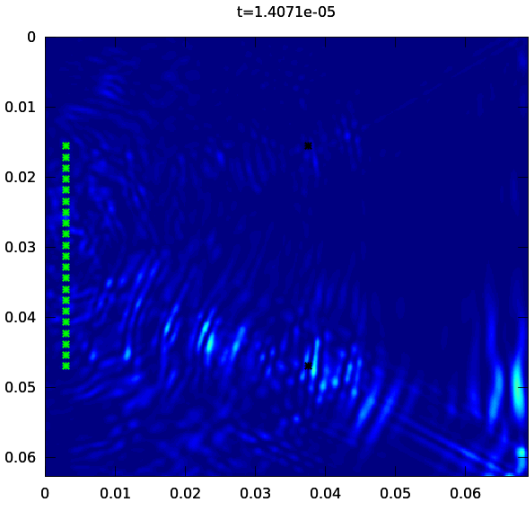
# Backward step - second singular value

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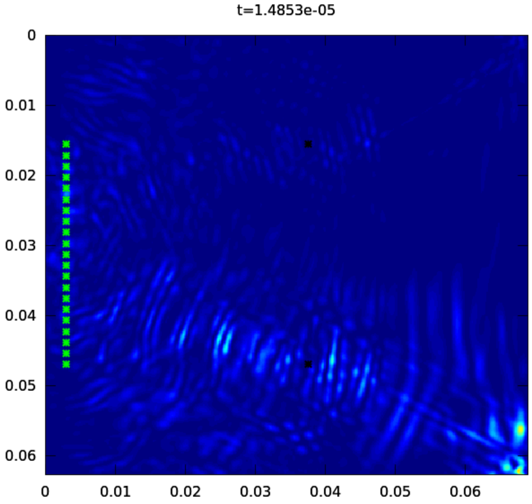
# Backward step - second singular value

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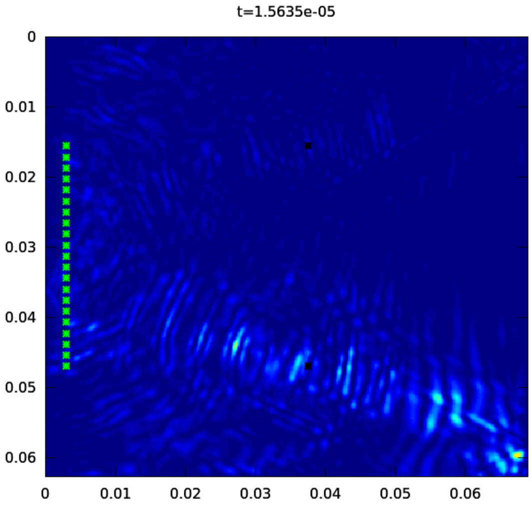
# Backward step - second singular value

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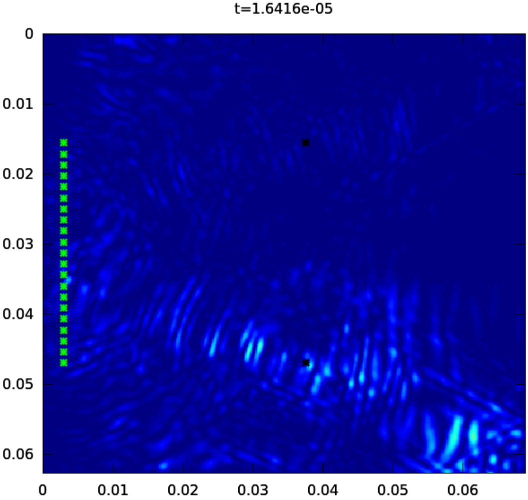
# Backward step - second singular value

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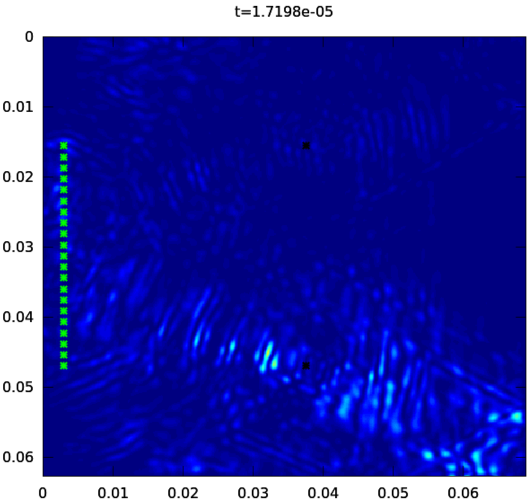
# Backward step - second singular value



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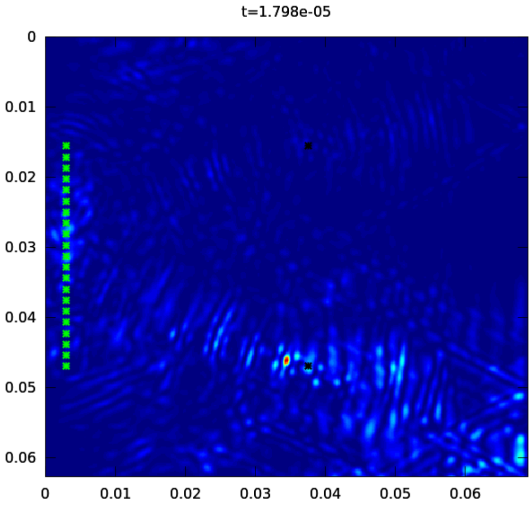


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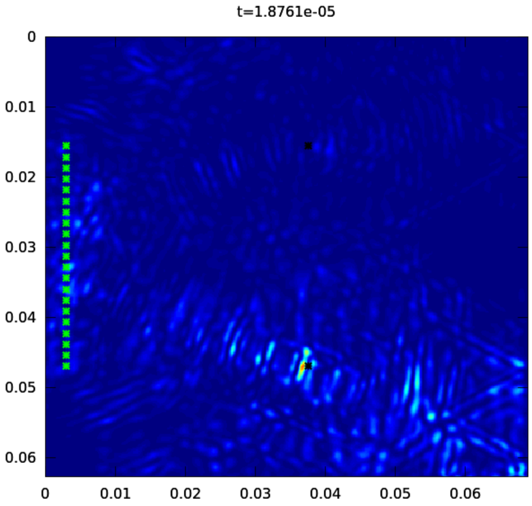
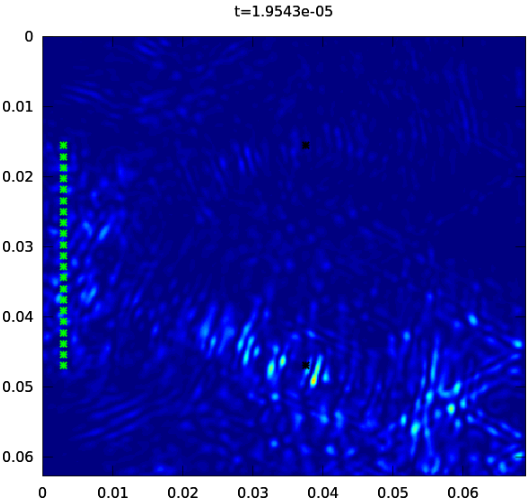
# Backward step - second singular value



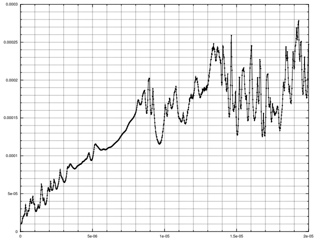

main

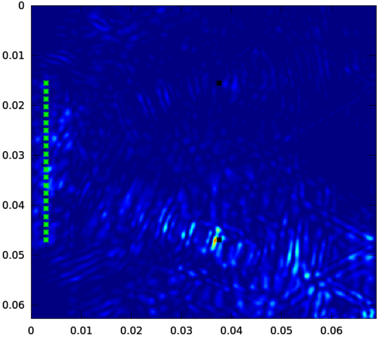
# Backward step - second singular value



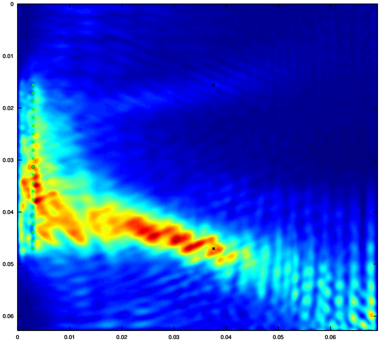

main

$$\mathcal{E}(t) = \frac{1}{2}(A\sigma, \sigma) + \frac{1}{2}(\rho v, v)$$



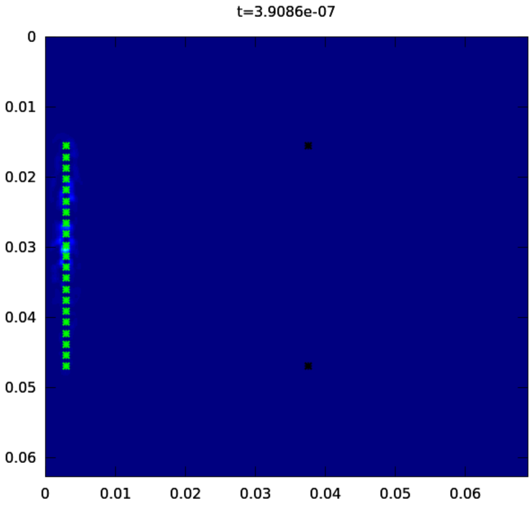



$$\mathcal{I}(\mathbf{y}) = \int_0^T \mathcal{E}(\mathbf{y}, t)\mathrm{d}t, \qquad \mathbf{y} \in \Omega$$




main

# Backward step - third singular value



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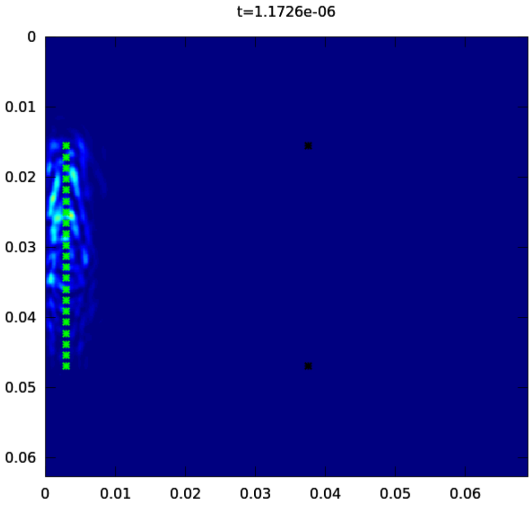

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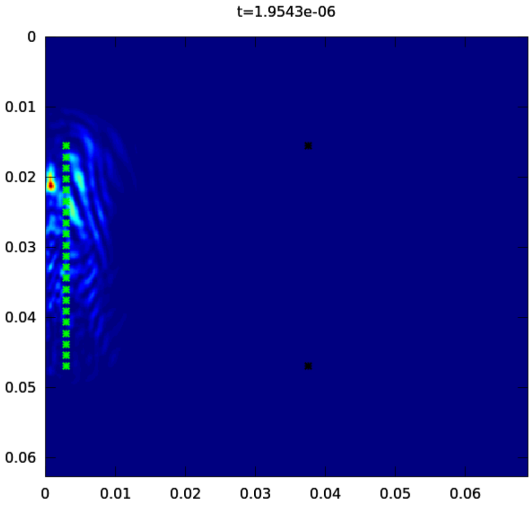


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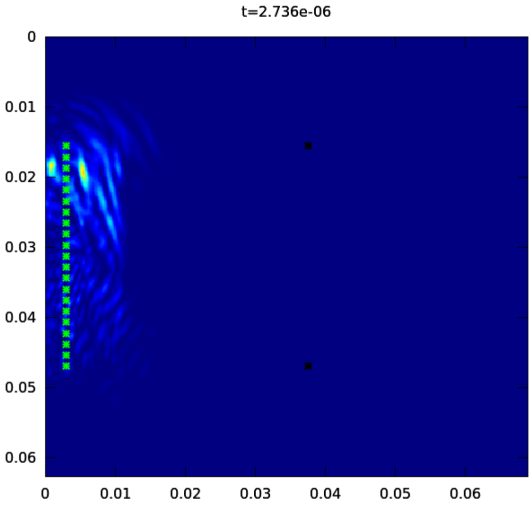


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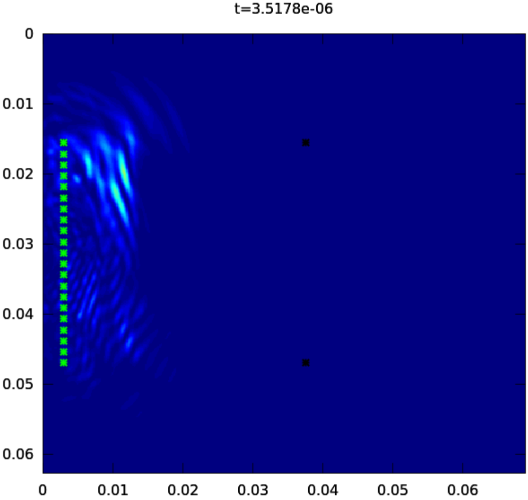


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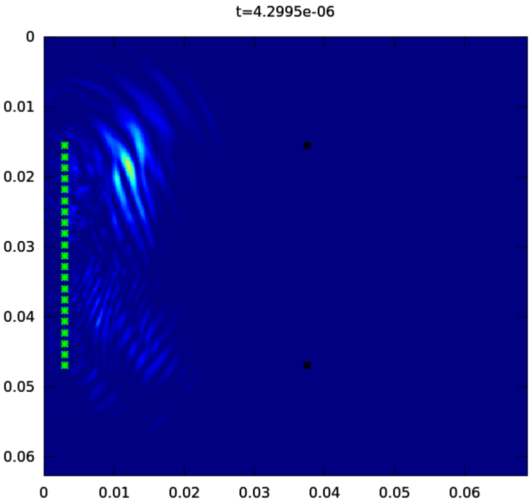
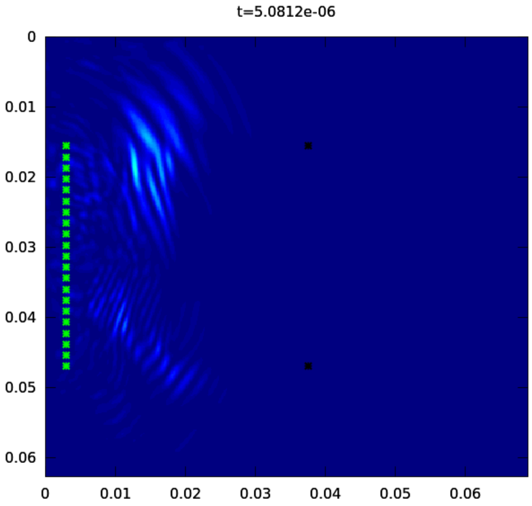



main

# Backward step - third singular value

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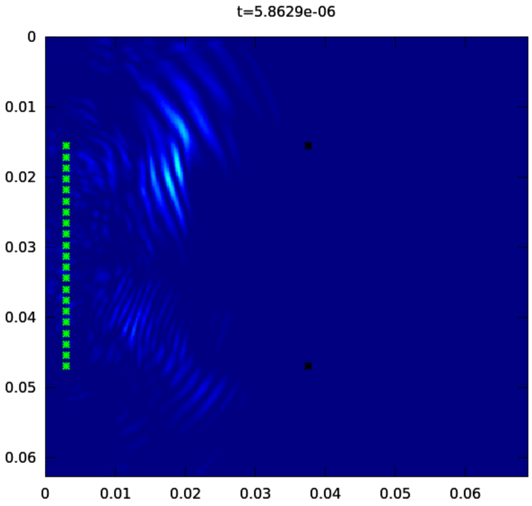


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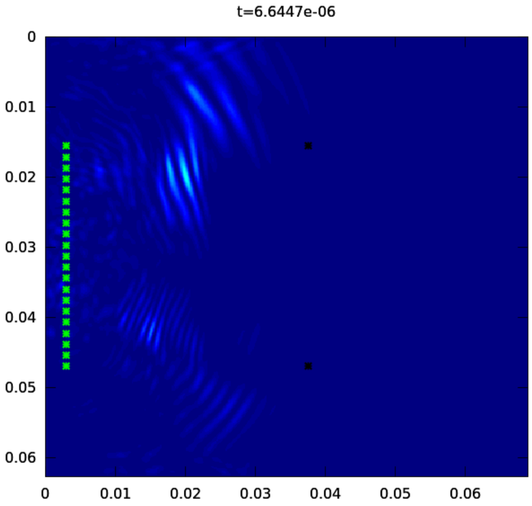


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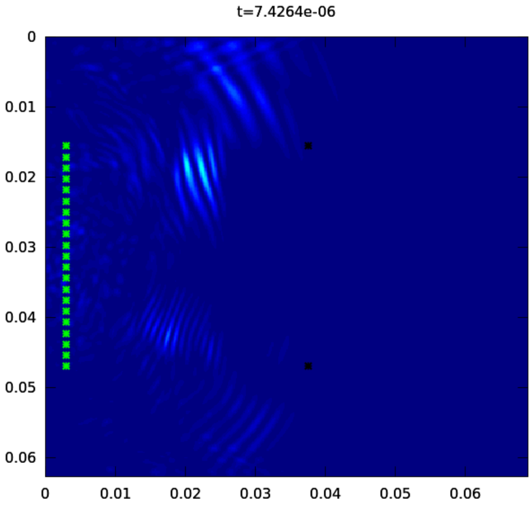
# Backward step - third singular value



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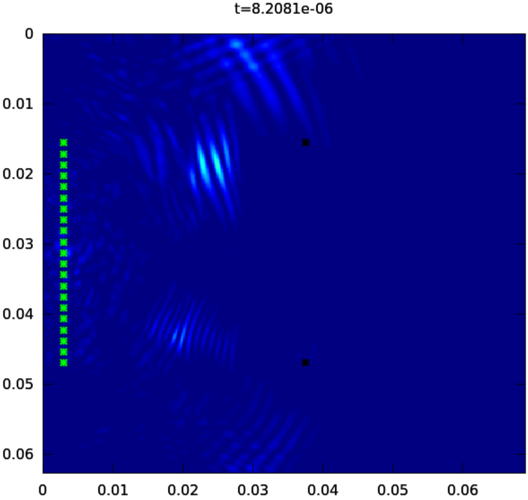


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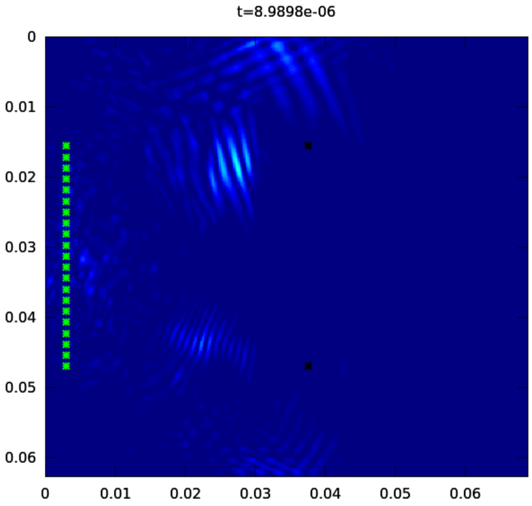


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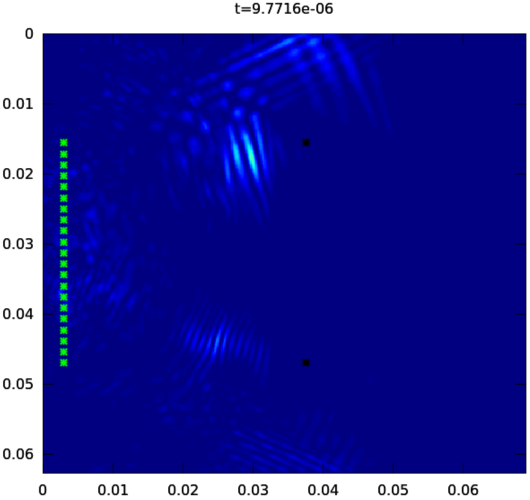


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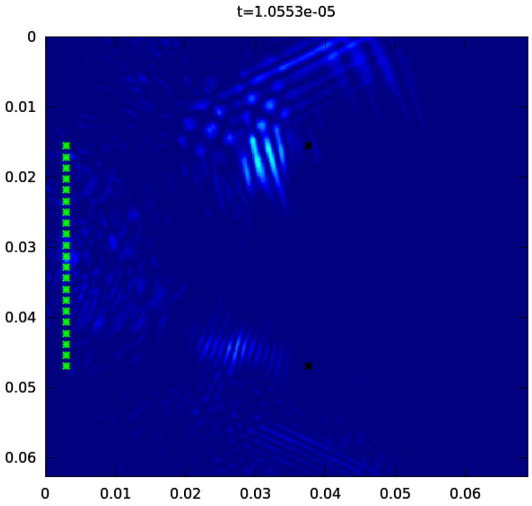
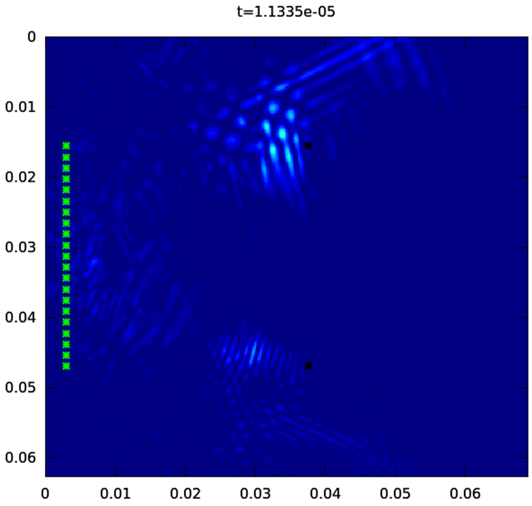
# Backward step - third singular value



# Backward step - third singular value



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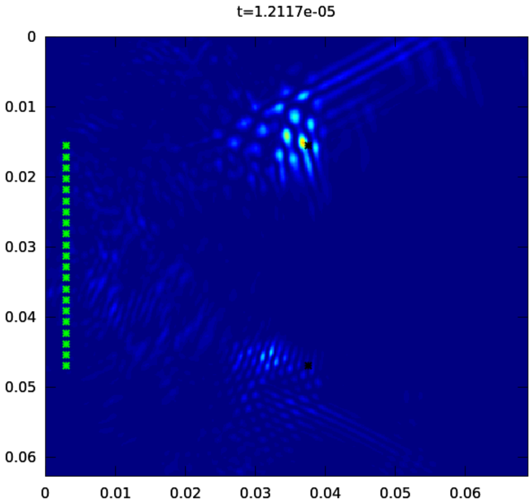


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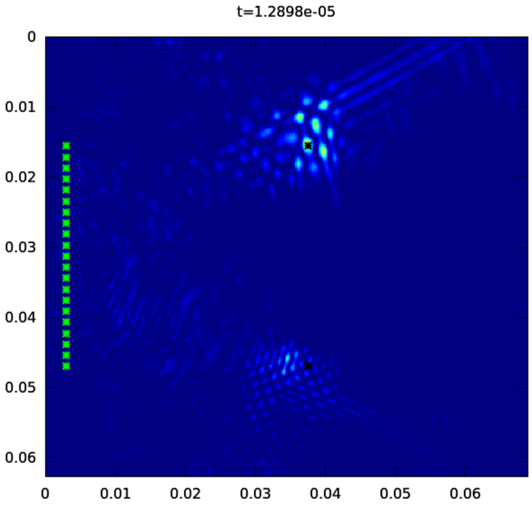
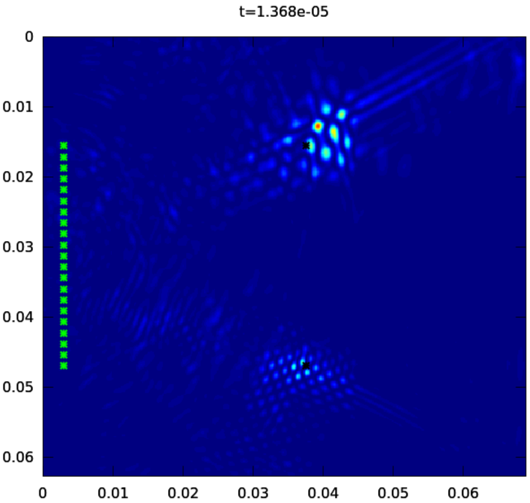


# Backward step - third singular value

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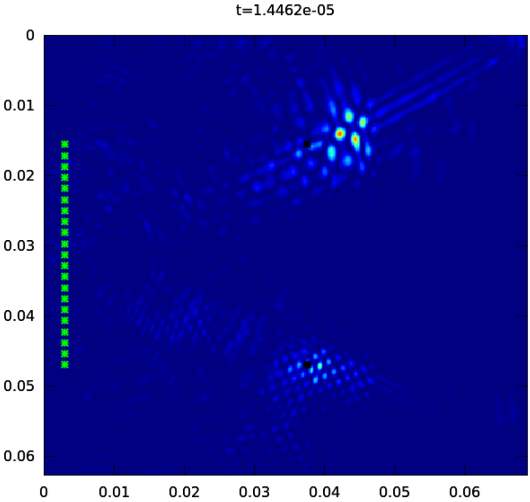


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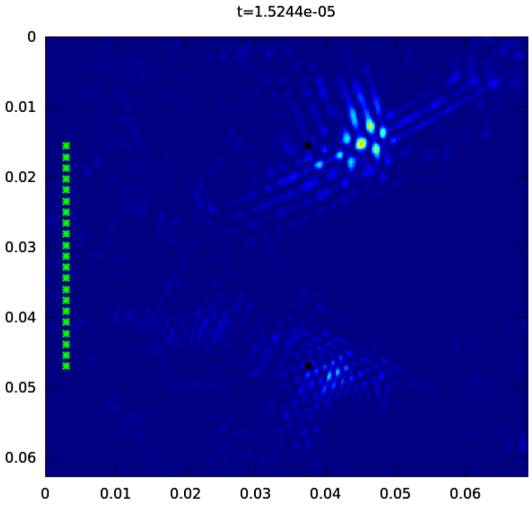

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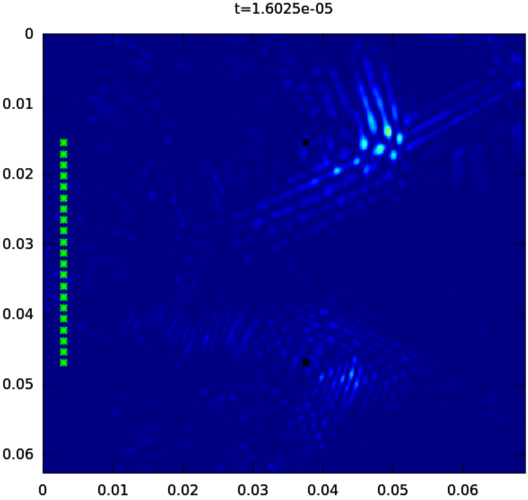


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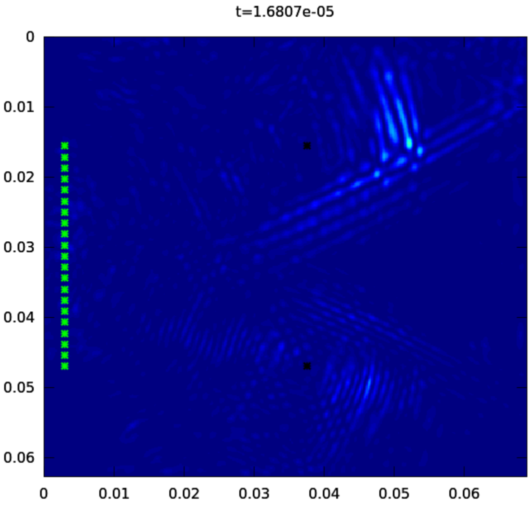
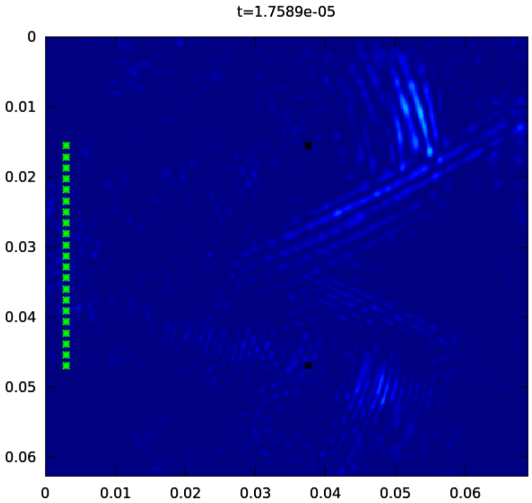
# Backward step - third singular value




main

# Backward step - third singular value

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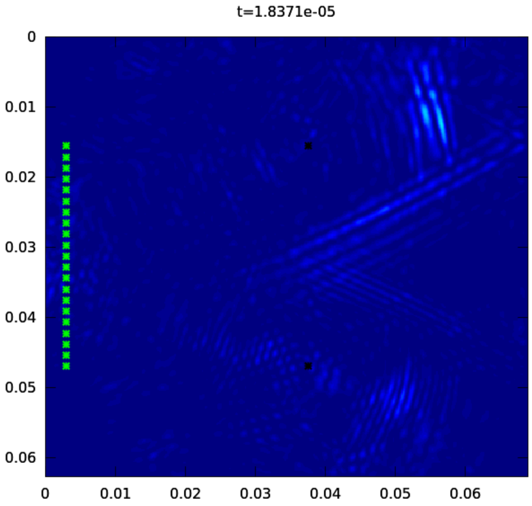
# Backward step - third singular value

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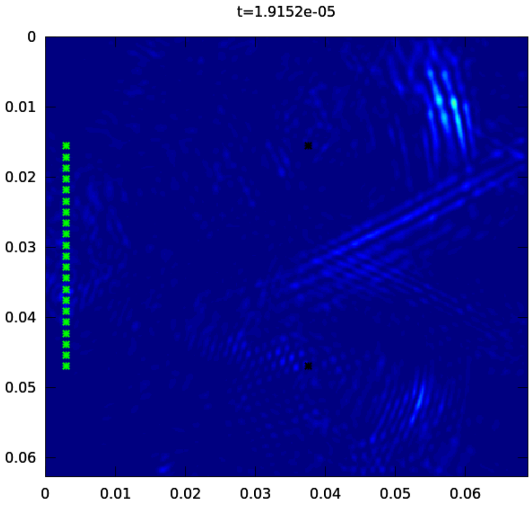


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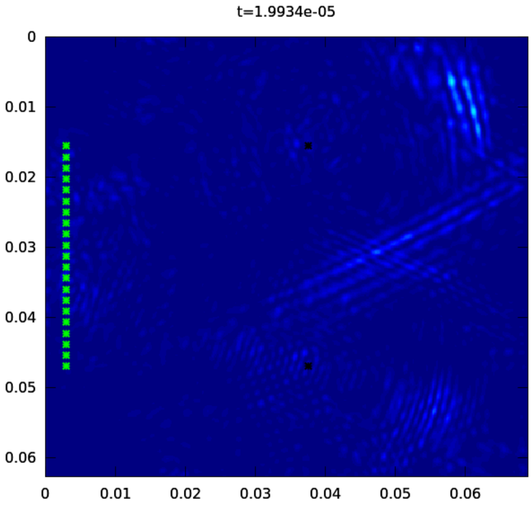
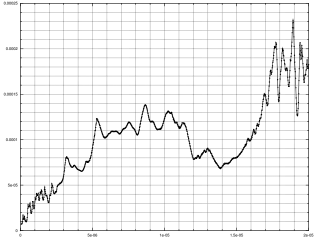
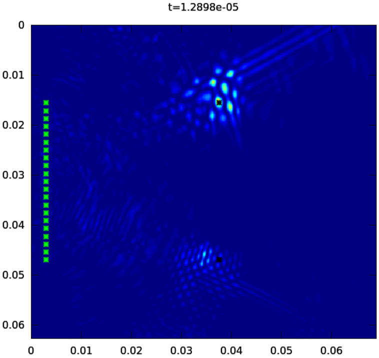
# Backward step - third singular value



$$\mathcal{E}(t) = \frac{1}{2}(A\sigma, \sigma) + \frac{1}{2}(\rho v, v)$$




$$\mathcal{I}(\mathbf{y}) = \int_0^T \mathcal{E}(\mathbf{y}, t)\mathrm{d}t, \qquad \mathbf{y} \in \Omega$$